A short note on the lower bounds for the Kirchhoff index of graphs

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Let G be a simple connected graph with $n \geq 2$ vertices, m edges and Laplacian eigenvalues $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n = 0$. The Kirchhoff index $Kf(G)$, of a simple connected graph is defined as [2]

$$Kf(G) = \sum_{i<j} r_{ij},$$

where r_{ij} is the effective resistance between the vertices i and j. A more appropriate formula from practical point of view, was put forward in [1] (see [6])

$$Kf(G) = n \sum_{i=1}^{n-1} \frac{1}{\mu_i}.$$

The topological index, later called general Randić index R_{-1}, is defined as [5]

$$R_{-1} = R_{-1}(G) = \sum_{i \sim j} \frac{1}{d_i d_j},$$

where $i \sim j$ denotes that vertices i and j are adjacent, and d_i denotes the degree of the vertex i.
In [3] (see also [4]) the following inequality was proved

$$Kf(G) \geq -1 + 2(n-1)R_{-1}. \tag{1}$$

In this paper we will prove the following inequality

$$Kf(G) \geq \frac{n^2(n-1) - m}{m} - 2(n-1)R_{-1}. \tag{2}$$

A comparison of the inequality (1) and (2), as well as the inequality (2) with other known inequalities for the lower bounds of $Kf(G)$ are considered.

References

[1] I. Gutman and B. Mohar, The quasi-Wiener and the Kirchhoff indices coincide, Journal of Chemical Information and Modeling **36** (1996), 982–985.

[2] D. J. Klein and M. Randić , Resistance distance, J. Math. Chem. **12** (1993), 81–95.

[3] I. Ž. Milovanović and E. I. Milovanović, On lower bounds for the Kirchhoff index,Kragujevac Journal of Science **39** (2017), 77–91.

[4] I. Ž. Milovanović and E. I. Milovanović, Bounds for the Kirchhoff and degree Kirchhoff indices, in: Bounds in Chemical Graph Theory-Mainstreams (I. Gutman, B. Furtula, K. C. Das, E. Milovanović, I. Milovanović, Eds), Mathematical Chemistry Monographs MCM 20, University of Kragujevac, Kragujevac, 2017, pp. 93–119.

[5] M. Randić, On characterization of molecular branching, Journal of the American Chemical Society **97** (1975), 6609–6615.

[6] H. Y. Zhu, D. J. Klein and I. Lukovits, Extensions of the Wiener number, Journal of Chemical Information and Computer Sciences **36** (1996), 420–428.