A short note on the lower bounds for the Kirchhoff index of graphs

Edin Glogić\(^1\), Emina Milovanović\(^2\), Igor Milovanović\(^2\), and Marjan Matejić\(^2\)

\(^1\)Department of Mathematics, State University of Novi Pazar, edinglogic@np.ac.rs
\(^2\)Faculty of Electronics Engineering, University of Niš, ema@elfak.ni.ac.rs, igor@elfak.ni.ac.rs, marjan.matejic@elfak.ni.ac.rs

Let \(G\) be a simple connected graph with \(n \geq 2\) vertices, \(m\) edges and Laplacian eigenvalues \(\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n = 0\). The Kirchhoff index \(Kf(G)\), of a simple connected graph is defined as \(\text{[2]}\)

\[
Kf(G) = \sum_{i<j} r_{ij},
\]

where \(r_{ij}\) is the effective resistance between the vertices \(i\) and \(j\). A more appropriate formula from practical point of view, was put forward in \([1]\) (see \([6]\))

\[
Kf(G) = n \sum_{i=1}^{n-1} \frac{1}{\mu_i}.
\]

The topological index, later called general Randić index \(R_{-1}\), is defined as \([5]\)

\[
R_{-1} = R_{-1}(G) = \sum_{i \sim j} \frac{1}{d_i d_j},
\]

where \(i \sim j\) denotes that vertices \(i\) and \(j\) are adjacent, and \(d_i\) denotes the degree of the vertex \(i\).

In \([3]\) (see also \([4]\)) the following inequality was proved

\[(1) \quad Kf(G) \geq -1 + 2(n-1)R_{-1}.\]

In this paper we will prove the following inequality

\[(2) \quad Kf(G) \geq \frac{n^2(n-1) - m}{m} - 2(n-1)R_{-1}.\]

A comparison of the inequality \((1)\) and \((2)\), as well as the inequality \((2)\) with other known inequalities for the lower bounds of \(Kf(G)\) are considered.
References


