Backward Euler and forward-backward Euler methods for pantograph stochastic differential equations under nonlinear growth conditions

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The main object of consideration is the implicit backward Euler method for a class of pantograph stochastic differential equations with coefficient which satisfy the generalized Khasminskii-type conditions. The one-sided Lipschitz condition on the drift coefficient is required in order to guarantee the existence and uniqueness of the backward Euler solution. In order to overcome some measurability difficulties, the forward-backward Euler method is employed. Under the conditions which are introduced, the convergence in probability on finite time intervals is established for the discrete and continuous forward-backward Euler solutions, as well as for discrete backward Euler solution. Moreover, under certain more restrictive nonlinear growth conditions it is proved that both discrete backward and forward-backward Euler solutions are globally a.s. asymptotically polynomially stable. The stability result is based on the application of the semimartingale convergence theorem. Numerical examples are provided to support the theoretical results.

References

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