Study of (σ, τ)-generalized derivations with their composition of semiprime rings

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The main purpose of this paper study and investigate some results concerning (σ, τ)-generalized derivations D associated with derivation d of semiprime ring and prime ring R, where σ and acts as two automorphism mappings of R. During this work, we suppose let R be an associative ring with the center of R which is denoted by $Z(R)$ and let σ, τ be automorphism mappings on R. We depend on the commutator $[x, y]_{\sigma,\tau} = x\sigma(y) - \tau(y)x$ (resp. $(xoy)_{\sigma,\tau} = x\sigma(y) + \tau(y)x)$ for all $x, y \in R$. Moreover, let $D : R \to R$ is an additive map and $d : R \to R$ is a derivation.

If $D(xy) = D(x)\sigma(y) + \tau(x)d(y)$ holds for all $x, y \in R$; then D is called a (σ, τ)-generalized derivation associated with d. We divided this paper into sections the preliminaries with some results contained in the first section while the second section we emphasis on composition of (σ, τ)-generalized derivations of the Leibniz's formula, where we introduce the general formula to computes the composition of (σ, τ)-generalized derivations and illustrated that by example. We supple some results about that where the σ and τ be two automorphism mappings of R such that their commute with D and d.

In fact, there are some applications of (σ, τ)-derivations which develop an approach to deformations of Lie algebras which have many applications in models of quantum phenomena and in analysis of complex systems. The map has been extensively investigated in pure algebra. Recently, it has been treated for the Banach algebra theory.

Following some results.

Theorem 1. *Let R be a 2-torsion free semiprime ring and σ and τ be two automorphism mappings of R. Suppose that there exists a (σ, τ)-generalized derivation D such that $[D(x), x]_{\sigma,\tau} = 0$ for all $x \in R$, then*

(i) *if the generalized derivation D commuting mapping of R then d is commuting mapping of R;*

(ii) *if the derivation d commuting mapping of R then $D$is 2-commuting mapping of R.*

Theorem 2. *Let n and r be a fixed positive integers. Let R be a 2-torsion free semiprime ring, σ and τ be two automorphism mappings of R such that the mappings σ and τ are commute with D and d, D a (σ, τ)-generalized derivation with an associated derivation d of R such that $[D^n(x), x^n]_{(\sigma,\tau)} = 0$, then*

$$\sum_{r=0}^{n} \binom{n}{r} D^{n-r}(x)d^r(x), x^{2n}] = -[D^n(x), x^{2n}]x \in Z(R), \quad \text{for all } x \in R.$$

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