Norm inequalities for a class of elementary operators

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Let $\sum_{n=1}^{\infty}\big(\|A_nh\|^2+\|A_n^*h\|^2+\|B_nh\|^2+\|B_n^*h\|^2\big) < +\infty$ for all h in a Hilbert space $\mathcal{H}$, for some families $\{A_n\}_{n=1}^{\infty}$ and $\{B_n\}_{n=1}^{\infty}$ of bounded operators on $\mathcal{H}$, where at least one of them consists of mutually commuting normal operators. For a symmetrically normed (s.n.) function Φ and $p \geqslant 2$, let $\Phi^{(p)^*}$ denote a s.n. function adjoint to p-modification $\Phi^{(p)}$ of Φ, then for all $X \in \mathcal{C}_{\Phi^{(p)*}}(\mathcal{H})$

$$\Big\|\sum_{n=1}^{\infty}A_nXB_n\Big\|_{\Phi^{(p)*}} \leqslant \Big\|\Big(\sum_{n=1}^{\infty}A_n^*A_n\Big)^{1/2}X\Big(\sum_{n=1}^{\infty}B_nB_n^*\Big)^{1/2}\Big\|_{\Phi^{(p)*}}.$$

Amongst other applications, this new Cauchy-Schwarz type norm inequality was used to explore a class of elementary operators induced by an analytic functions with non-negative Taylor coefficients to prove that

$$\Big\|f\Big(\sum_{n=1}^{\infty}A_n\otimes B_n\Big)X\Big\|_{\Phi^{(p)*}} \leqslant \Big\|\sqrt{f\Big(\sum_{n=1}^{\infty}A_n^*\otimes A_n\Big)(I)}\,X\sqrt{f\Big(\sum_{n=1}^{\infty}B_n\otimes B_n^*\Big)(I)}\Big\|_{\Phi^{(p)*}},$$

where $A_n\otimes B_n$ stands for the bilateral multipliers $A_n\otimes B_n\colon \mathcal{B}(\mathcal{H})\to\mathcal{B}(\mathcal{H})\colon X\mapsto A_nXB_n$. Different applications and examples for the obtained norm inequalities are also provided.

References

[1] D. R. Jocić, M. Lazarević and S. Milošević, Norm inequalities for a class of elementary operators generated by analytic functions with non-negative Taylor coefficients in ideals of compact operators related to p-modified unitarily invariant norms, Linear Alg. Appl. **540** (2018), 60–83.