Chessboard complex and its generalizations

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The *chessboard complex* $\Delta_{n,m}$ can be defined as the simplicial complex whose faces are all non-taking rook placements (no two rooks on the same row or column) on a $m \times n$ "chessboard". This simplicial complex appears in many interesting combinatorial situations, and some of its topological properties (connectivity, the structure of an orientable pseudomanifold) played the fundamental role in the proof of some interesting non-trivial results (colored Tverberg Theorem). There are some natural generalizations of a chessboard complex:

- Chessboard complex on a triangular board $\Psi_{a_n,\dots,a_1}$ (a left justified board with a_i rows of length i);
- Multiple chessboard complex $\Delta_{m,n}^{k_1,\dots,k_n;l_1,\dots,l_m}$ (at most k_i rooks in the i-th row and at most l_j in the j-th column);
- Symmetric multiple chessboard complex $\Sigma_{m,n}^{k_1,\dots,k_n;\mathbf{1}} := \bigcup_{\pi\in G} \Delta_{m,n}^{k_{\pi(1)},\dots,k_{\pi(n)};1,1,\dots,1}$.

We use standard combinatorial tools (shellability and discrete Morse theory) to investigate some topological properties of these complexes. These complexes naturally appear as appropriate configuration spaces for problems of Tverberg type, and improved estimates of their connectivity often leads to new results. Also, we will show that an "optimal multiple chessboard complex" can be naturally interpreted as a relative and a generalization of Bier spheres.

This talk is based on the joint work with S. Vrećica and R. Živaljević.