On the geometric-arithmetic index

Milica Milivojević[1] and Ljiljana Pavlović[1]

[1]Department of Mathematics, Faculty of Science, University of Kragujevac,
milica.milivojevic.88@gmail.com, pavlovic@kg.ac.rs

Abstract. Let $G(k,n)$ be the set of connected simple n-vertex graphs with minimum vertex degree k. The geometric-arithmetic index $GA(G)$ of a graph G is defined by $GA(G) = \sum_{uv} \frac{2\sqrt{d_u d_v}}{d_u+d_v}$, where $d(u)$ is the degree of vertex u and the summation extends over all edges uv of G. In this paper we characterized graphs on which GA index attains minimum value, when number of vertices of degree k is $n-1$ and $n-2$. We also gave a conjecture about the extremal graphs on which this index attains its minimum value and lower bound for this index for graphs with given minimum degree k, where $k \leq \lfloor k_0 \rfloor$, $k_0 = q_0(n-1)$, $q_0 \approx 0.0874$ is the unique positive root of equation $q\sqrt{q} + q + 3\sqrt{q} - 1 = 0$.

References

[1] G. Caporossi and I. Gutman, P. Hansen, Variable neighborhood search for extremal graphs. 4. Chemical trees with extremal connectivity index, Computers & Chemistry **23** (1999), 469 –477.

[2] K. Das, I. Gutman and B. Furtula, Survey on Geometric-Arithmetic Indices of Graphs, MATCH Commun. Math. Comput. Chem. **65** (2011), 595–644.

[3] T. Divnić, M. Milivojević and Lj. Pavlović, Extremal graphs for the geometric-arithmetic index with given minimum degree, Discrete Appl. Math. **162** (2014), 386–390.

[4] Z. Du, B. Zhou and N. Trinajsitć, On Geometric-Arithmetic Indices of (Molecular) Trees, Unicyclic Graphs and Bicyclic Graphs, MATCH Commun. Math. Comput. Chem. **66** (2011), 681–697.

[5] G. Fath-Tabar, B. Furtula and I. Gutman, A new geometric-arithmetic index, J. Math. Chem. **47**(1) (2010), 477–486.

[6] M. Mogharrab and G. Fath-Tabar, Some Bounds on GA_1 Index of Graphs, MATCH Commun. Math. Comput. Chem. **65** (2011), 33–38.

[7] J. M. Rodríguez and J. M. Sigarreta, On the Geometric-Arithmetic Index, MATCH Commun. Math. Comput. Chem. **74** (2015), 103–120.

[8] [22] M. Sohrabi-Haghighat and M. Rostami, Using linear programming to find the extremal graphs with minimum degree 1 with respect to Geometric-Arithmetic index, Applied Mathematics in Engineering, Management and Technology **3**(1) (2015), 534–539.

[9] D. Vukičević and B. Furtula, Topological index based on the ratios of geometrical and arithmetical means of end -vertex degrees of edges, J. Math. Chem. **46**(2) (2009), 1369–1376.

[10] Y. Yuan, B. Zhou and N. Trinajstić, On geometric-arithmetic index, J. Math. Chem. **47**(2) (2010), 833–841.

[11] B. Zhou, I. Gutman, B. Furtula and Z. Du, On two types of geometric-arithmetic index, Chemical Physics Letters **482** (2009), 153–155.