

Applications of Lyapunov functions to Caputo fractional differential equations

Snezhana Hristova¹, Ravi Agarwal², and Donal O'Regan³

¹Department of Applied Mathematics, Faculty of Mathematics and Informatics, University of Plovdiv, Plovdiv, Bulgaria, snehri@gmail.com

²Department of Mathematics, Texas A&M University-Kingsville, Kingsville, TX 78363, USA, agarwal@tamuk.edu

³School of Mathematics, Statistics and Applied Mathematics, National University of Ireland, Galway, Ireland, donal.oregan@nuigalway.ie

One approach to study various stability properties of solutions of nonlinear Caputo fractional differential equations is based on using Lyapunov like functions. A basic question which arises is the definition of the derivative of the Lyapunov like function along the given fractional equation. In this paper several definitions known in the literature for the derivative of Lyapunov functions among Caputo fractional differential equations are given. Applications and properties are discussed. The purpose of this paper is to refine the fundamental theorems and to discuss and illustrate some of these results and to present some new ones. A Caputo fractional Dini derivative of a Lyapunov function among nonlinear Caputo fractional differential equations is presented. Comparison results using this definition and scalar fractional differential equations are presented and several sufficient conditions for stability and asymptotic stability with respect to part of the variables are given. Several examples are given to illustrate the theory.

Acknowledgments. Research was partially supported by the Fund NPD, Plovdiv University, No. MU17-FMI-007.