

Automated proving of some inequalities involving trigonometric, inverse trigonometric and exponential functions

Branko Malešević¹, Tatjana Lutovac¹, Marija Rašajski¹, and Bojan Banjac²

¹Department of Applied Mathematics, School of Electrical Engineering, University of Belgrade
branko.malesevic@etf.bg.ac.rs, tatjana.lutovac@etf.bg.ac.rs, marija.rasajski@etf.bg.ac.rs

²PhD student of Applied Mathematics, School of Electrical Engineering, University of Belgrade,
Computer graphics chair, Faculty of Technical Sciences, University of Novi Sad
bojan.banjac@uns.ac.rs

Trigonometric, inverse trigonometric and exponential functions play an important role and have applications in engineering and in many areas of pure and applied science. In particular, various inequalities related to the above-mentioned functions have been studied and effectively applied to problems in pure science and many areas of engineering, such as telecommunications, electronics and aeronautics. Estimates, stemming from these inequalities, are particularly useful in approximation theory, Fourier and Harmonic Analysis.

Proving these kinds of inequalities by hand, if possible, is usually complicated. Also, solving these types of problems proves to be an error-prone task involving complex and demanding computations.

Various techniques and provers, both interactive and automated, have been developed in the last decade. However, existing provers are not suitable for direct applications by users not familiar with their design and the underlying formal system.

In this work we present some of the inequalities and automated techniques that we worked on and developed. These techniques involve power-series expansions, some recently developed estimates related to the Taylor polynomials, as well as some applications of existing theorems about analytic functions (see [1-15]).

Our approach and solutions prove to be efficient and easy to follow, while staying mathematically rigorous. Also, in many cases we were able to estimate the error of approximation, as well as to generate new families of inequalities.

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