Weyl asymptotic formulas for infinite order $\mathbf{\Psi}$DOs and Sobolev type spaces

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I will present results on the Weyl asymptotic formulae for the operators that are not of power-log-type as in the finite order (distributional) setting, but of log-type, which in turn yields that the eigenvalues of infinite order ΨDO, with appropriate assumptions, are "very sparse". The heat kernel analysis needed for the proofs of the Weyl asymptotic formulae for the class of operators is based on the complex powers of hypo-elliptic type ΨDO of infinite order. In this way, we obtain the semigroup $T(t)f = \sum_{j=0}^{\infty} e^{-t\lambda_j}(f, \varphi_j)\varphi_j$, $f \in L^2(\mathbb{R}^d)$, $t \geq 0$, with the infinitesimal generator $-\overline{A}$ (the closure of $-a^w$ in $L^2(\mathbb{R}^d)$) where λ_j and φ_j are the eigenvalues and eigenfunctions of $\overline{A}$; a^w is the Wayl operator for the symbol a.

Infinite order Sobolev type spaces $H^*_{A_p,\rho}(f)$, where the order is given by a functions f belonging to a certain class of "admissible" functions of sub-exponential (i.e. ultrapolynomial) growth. $H^*_{A_p,\rho}(f)$ satisfies most of the familiar results for the classical, finite order, Sobolev spaces. Moreover, I will present the Fredholm properties of infinite order ΨDOs having hypoelliptic symbols satisfying elliptic bounds with respect to an admissible function f.

The talk is based on collaborative works with Bojan Prangoski and Jasson Vindas.