On the notions of structure species in the senses of Tsalenko, Sonner, and Blanchard

Aslanbek Naziev

Department Math&MathTeach, Faculty Physics&Mathematics, Ryazan State University, RU, a.naziev@rsu.edu.ru

Blanchard [1] introduced the concept of structure species on a category very close to the concept of structure species in the sense of Bourbaki [2], and proved that the concept introduced by him is equivalent to the concept of structure species in the sense of Sonner [3]. According to Sonner, a structure species on a category $\mathcal{X}$ is an univalent functor $F\colon \mathcal{Y} \to \mathcal{X}$ such that for any object Y of the category $\mathcal{Y}$ and any isomorphism $j\colon F(Y) \to$ *??* into $\mathcal{X}$ there exists an isomorphism $i\colon Y \to$ *?* in $\mathcal{Y}$ such that $j = F(i)$. Finally, Tsalenko [4] introduced the notion of structured category over the category $\mathcal{X}$, meaning by this the ordered pair $(\mathcal{Y},\ F)$, formed by the category $\mathcal{Y}$ and univalent functor F from $\mathcal{Y}$ to $\mathcal{X}$. It is clear that if $F\colon \mathcal{Y} \to \mathcal{X}$ is the structure species in Sonner sense, then $(\mathcal{Y},\ F)$ is a structured category over $\mathcal{X}$ in the sense of Tsalenko, and that the converse, generally speaking, is incorrect. Nevertheless, the following theorem holds.

Theorem 0.1. *Let $F\colon \mathcal{Y} \to \mathcal{X}$ be an univalent functor. Then there exist the category $\hat{\mathcal{Y}}$, the structure species (in the sense of Blanchard) $\hat{F}\colon \hat{\mathcal{Y}} \to \mathcal{X}$ and the equivalence $(U,\ V)$ between the categories $\mathcal{Y}$ and $\hat{\mathcal{Y}}$ such that $\hat{F} \circ U = F$ and $V \circ U = 1_{\mathcal{X}}$.*

Corollary 0.1. *Let $\mathcal{Y}$ be a category with a generator (respectively, with a cogenerator). Then there is the structure species Σ on the category of sets such that the category $\mathcal{Y}$ (respectively, $\mathcal{Y}^{\mathrm{op}}$, the dual to $\mathcal{Y}$) is equivalent to the category of all Σ-objects and Σ-morphisms.*

References

[1] A. Blanchard, Structure species and constructive functors, Canad. J. Math. **26**(5) (1974), 1217–1227.

[2] N. Bourbaki, Éléments de Mathématique, Livre I, Theorie des Ensembles, Chapitre IV, Structures, Hermann, Paris, 1957.

[3] J. Sonner, Lifting unductive and projective limits, Canad. J. Math. **19**(5) (1967), 1329–1339.

[4] M. Sh. Tsalenko, Functors between structurized categories, Mat. Sb. (N.S.) **80(122)** 4(12) (1969), 533–552.