Hilbert matrix on mixed norm spaces

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We prove that the Hilbert matrix operator $H$ is bounded on the mixed norm space $H_{\nu}^{p,q,\alpha}$ if and only if $0 < \kappa_{p,\alpha,\nu} < 1$, where $\kappa_{p,\alpha,\nu} = \nu - \alpha - \frac{1}{p} + 1$. In particular, Hilbert matrix operator $H$ is bounded on the weighted Bergman space $A^{p,\alpha}$ if and only if $1 < \alpha + 2 < p$ and it is bounded on the Dirichlet space $D_p^\alpha$ if and only if $\max\{-1, p-2\} < \alpha < 2p - 2$. Also, it is well known that the norm of the Hilbert matrix operator $H$ on the Bergman space $A^p$ is equal to $\frac{\pi}{\sin \frac{2\pi}{p}}$, when $4 \leq p < \infty$, and it was also conjectured that

$$
\|H\|_{A^p \to A^p} = \frac{\pi}{\sin \frac{2\pi}{p}},
$$

when $2 < p < 4$. Following [1] we prove this conjecture.

References

