Geodesically equivalent metrics on homogenous spaces

Neda Bokan[1]

[1]Faculty of Mathematics, University of Belgrade, Serbia, neda@matf.bg.ac.rs

Two metrics on a manifold are geodesically equivalent if sets of their unparameterized geodesics coincide. In this paper we show that if two left G-invariant metrics of arbitrary signature on homogenous space G/H are geodesically equivalent, they are affinely equivalent, i.e. they have the same Levi-Civita connection. We also prove that existence of non-proportional, geodesically equivalent, G-invariant metrics on homogenous space G/H implies that their holonomy algebra cannot be full. We give an algorithm for finding all left invariant metrics geodesically equivalent to a given left invariant metric on a Lie group. Using that algorithm we prove that no two left invariant metric, of any signature, on sphere S^3 are geodesically equivalent. However, we present examples of Lie groups that admit geodesically equivalent, non-proportional, left-invariant metrics.

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