Billiards within quadrics, Riemann surfaces, isomonodromy deformations, and extremal polynomials

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We will review recent and new results on billiards within confocal quadrics and their dynamical, geometric, and arithmetic properties. By connecting these questions with the analysis on Riemann surfaces, in particular elliptic and hyperelliptic curves, we construct solutions to the Painlevé VI and Schlesinger equations on isomonodromy deformations. We map the billiard dynamics within confocal conics to rectangular billiard dynamics, which leads to a novel concept in the ergodic theory, "the genericity along the curves" (Fraczek, Shi, Ulcigrai). By developing a bridge toward the theory of extremal polynomials and Pell's equations, we derive fundamental properties of the billiard dynamics, winding numbers and frequency map. As an application, we provide a detailed description of periodic trajectories in an arbitrary dimension d with small periods T, $d \leq T \leq 2d$, emphasizing the cases $d = 3$, $d = 4$. In part, the results are joint with Milena Radnović and in part with Vasilisa Shramchenko. The results are obtained as parts of grants 174020 MPNTR and NSF 1444147.

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