On Calabi-Yau representatives in the SU-bordism ring

Ivan Limonchenko[1], Zhi Lü[1], and Taras E. Panov[2]

[1]School of Mathematical Sciences, Fudan University, ilimonchenko@fudan.edu.cn, zlu@fudan.edu.cn
[2]Department of Higher Geometry and Topology, Faculty of Mechanics and Mathemetics, Moscow State University, tpanov@mech.math.msu.su

To find nice geometric representatives of bordism classes and bordism ring generators for various bordism theories has been a classical problem in algebraic and differential topology since 1960s. From that time till nowadays much was done regarding this problem, starting with the influential works by Conner, Floyd, Milnor, Novikov, Stong, and others. In 1958 F. Hirzebruch stated a problem, which remains open until now: to find a nonsingular (connected) complex algebraic variety in a given unitary bordism class. It was proved in 1960s that Milnor hypersurfaces generate the unitary bordism ring over integers, which is a polynomial ring due to a classical result of J. Milnor and S. P. Novikov, and similar generators also exist for unoriented and oriented bordism rings.

In 1962 S. P. Novikov proved that the special unitary bordism ring over integers with 2 reversed is isomorphic to a polynomial ring with one generator in each even real dimension greater than two. Z. Lü and T. E. Panov (2014) constructed a quasitoric representative for each multiplicative generator of this ring, starting with real dimension 10; quasitoric manifolds represent zero in dimensions 4, 6, and 8.

In this talk we are going to discuss Hirzebruch problem for SU-bordism. J.Mosley (2016) proved that a nonsingular complex algebraic variety may not exist in a given SU-bordism class already in dimension 4. However, we show [1] that for each multiplicative generator in the SU-bordism ring such a representative (disconnected in general) can be found using V. V. Batyrev's construction (1993) of Calabi–Yau hypersurfaces in toric Fano varieties over reflexive polyhedra.

This is a joint work with Zhi Lü (Fudan University) and Taras E. Panov (Moscow State University).

References

[1] I. Limonchenko, Z. Lü and T. E. Panov, Calabi-Yau hypersurfaces and SU-bordism, (2017), arXiv:1712.07350, (preprint).