

Algebraic geometrical description of Neumann systems on Stiefel varieties

Božidar Jovanović¹

¹Mathematical Institute SANU, Belgrade, bozaj@mi.sanu.ac.rs

We study geometric and algebraic geometric properties of the continuous and discrete Neumann systems on cotangent bundles of Stiefel varieties $V_{n,r}$. The systems are integrable in the non-commutative sense, and by applying a $2r \times 2r$ -Lax representation, we show that generic complex invariant manifolds are open subsets of affine (non-compact) Prym varieties on which the complex flow is linear. The characteristics of the varieties and the direction of the flow are calculated explicitly. Next, we construct a family of (multi-valued) integrable discretizations of the Neumann systems and describe them as translations on the Prym varieties, which are written explicitly in terms of divisors of points on the spectral curve. It appears that the systems inherit or naturally generalize the basic properties of the classical Neumann system on S^{n-1} and, therefore, of the Jacobi–Mumford systems: the structure of the Lax matrices, the spectral curve, the equations of motion, linearization on Abelian varieties, and, in the discrete case, the formula for the translation on them. The results are obtained in a collaboration with Yuri Fedorov.

References

- [1] Yu. Fedorov and B. Jovanović, Geodesic Flows and Neumann Systems on Stiefel Varieties: geometry and integrability, *Math. Z.* **270**(3-4) (2012), 659–698, arXiv:1011.1835.
- [2] Yu. Fedorov, B. Jovanović, Continuous and discrete Neumann systems on Stiefel varieties as matrix generalizations of the Jacobi-Mumford systems, arXiv:1503.07053.