Orthogonal shadows and index of Grassmann manifolds

Djordje Baralić[1], Pavle V. M. Blagojević[1], Roman Karasev[2], and Aleksandar Vučić[3]

[1]Mathematical Institute SASA, Belgrade, Serbia, djbaralic@mi.sanu.ac.rs, blagojevic@math.fu-berlin.de
[2]Institute for Information Transmission Problems RAS, Moscow, Russia, r_n_karasev@mail.ru
[3]University of Belgrade, Faculty of Mathematics, Belgrade, Serbia, avucic@matf.bg.ac.rs

In this paper we study the $\mathbb{Z}/2$ action on real Grassmann manifolds $G_n(\mathbb{R}^{2n})$ and $\widetilde{G}_n(\mathbb{R}^{2n})$ given by taking (appropriately oriented) orthogonal complement. We completely evaluate the related $\mathbb{Z}/2$ Fadell–Husseini index utilizing a novel computation of the Stiefel–Whitney classes of the wreath product of a vector bundle. These results are used to establish the following geometric result about the orthogonal shadows of a convex body: For $n = 2^a(2b+1)$, $k = 2^{a+1} - 1$, C a convex body in $\mathbb{R}^{2n}$, and k real valued functions $\alpha_1, \dots, \alpha_k$ continuous on convex bodies in $\mathbb{R}^{2n}$ with respect to the Hausdorff metric, there exists a subspace $V \subseteq \mathbb{R}^{2n}$ such that projections of C to V and its orthogonal complement $V^\perp$ have the same value with respect to each function α_i, which is $\alpha_i(p_V(C)) = \alpha_i(p_{V^\perp}(C))$ for all $1 \le i \le k$.