On applications of Algebraic topology (Chessboard complexes)

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The results of Algebraic topology were successfully applied in establishing many very important results in different areas of Mathematics, such as Fundamental theorem of algebra, Brouwer fixed point theorem (which was called Fundamental theorem of analysis when appeared), the ham sandwich theorem, the proof of Kneser conjecture etc.

We illustrate the applicability of topological methods and results by presenting an important configuration space - chessboard complex, and by showing how its properties could be used in solving the problems in other areas of Mathematics. Chessboard complex appears in different ways: as a coset complex of the symmetric group by some of its subgroups (stabilizing some elements), as a matching complex of a complete bipartite graph, as a complex of partial injective functions from one finite set to the other, as a deleted join of a finite set.

Actually, we define several versions of this complex and show how each of them is motivated by some mathematical question. For example, we show how a cycle-free chessboard complex appears in establishing the symmetric analogue of the cyclic homology of algebras, and how generalized and symmetrized versions appear in establishing the generalizations of van Kampen-Flores theorem and Tverberg-type theorems.

Our dominant interest is in the connectivity properties of a chessboard complex (which reduces to determining its homology groups), but we consider some other properties as well.

The talk is based on joint papers with R. Živaljević, and some with D. Jojić.