Cyclohedron and Kantorovich-Rubinstein polytopes

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We show that the cyclohedron (Bott-Taubes polytope) W_n arises as the polar dual of a Kantorovich-Rubinstein polytope $KR(\rho)$, where ρ is an explicitly described quasi-metric (asymmetric distance function) satisfying strict triangle inequality. From a broader perspective, this phenomenon illustrates the relationship between a nestohedron $\Delta_{\widehat{\mathcal{F}}}$ (associated to a building set $\widehat{\mathcal{F}}$) and its non-simple deformation $\Delta_{\mathcal{F}}$, where $\mathcal{F}$ is an *irredundant* or *tight basis* of $\widehat{\mathcal{F}}$ (([2, Definition 21])). Among the consequences are a new proof of a recent result of Gordon and Petrov (Arnold Math. J. **3** (2) (2017), 205–218) about f-vectors of generic Kantorovich-Rubinstein polytopes and an extension of a theorem of Gelfand, Graev, and Postnikov, about triangulations of the type A, positive root polytopes.

References

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