

Numerical solution of parabolic-hyperbolic transmission problem

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Layers with material properties which significantly differ from those of the surrounding medium appear in a variety of applications. The layer may have a structural role (as in the case of glue), a thermal role (as in the case of a thin thermal insulator), an electromagnetic or optical role etc. Mathematical models of energy and mass transfer in domains with layers lead to so called interface or transmission problems. In this paper we consider a class of non-standard parabolic-hyperbolic transmission problem in disjoint domains. As a model example we take an area consisting of two non-adjacent rectangles. In each rectangle an Robin’s initial-boundary value problem is given. The interaction between their solutions is described using nonlocal integral conjugation conditions Robin-Dirichlet type on the boundaries of the observed subareas. For the model problem the existence and uniqueness of its weak solution in appropriate Sobolev-like space is proved. A finite difference scheme approximating this problem is proposed and analyzed. An estimate of the convergence rate, compatible with the smoothness of the input data is obtained.

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