Some numerical radius and norm inequalities in Hilbert space operators

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Let $\mathcal{B}(\mathcal{H})$ denote the C^*-algebra of all bounded linear operator on a complex Hilbert space $\mathcal{H}$ with inner product $\langle \cdot, \cdot \rangle$. For $A \in \mathcal{B}(\mathcal{H})$ let $\omega(A) = \sup\{|\langle x, Ax\rangle| : \|x\| = 1\}$, $\|A\| = \sup\{\|Ax\| : \|x\| = 1\}$ and $|A| = (A^*A)^{1/2}$ denote the numerical radius, the usual operator norm of A and the absolute value of A, respectively. It is well know that $\omega(\cdot)$ is a norm on $\mathcal{B}(\mathcal{H})$, and that for all $A \in \mathcal{B}(\mathcal{H})$,

$$\frac{1}{2}\|A\| \leq \omega(A) \leq \|A\|.$$

It is shown that, if $A \in \mathcal{B}(\mathcal{H})$ is a hyponormal operator. Then,

$$\omega(A) \leq \frac{1}{2\left(1 + \frac{\xi_{|A|}^2}{8}\right)} \left\| |A| + |A^*| \right\|,$$

where $\xi_{|A|} = \inf_{\|x\|=1} \left\{ \frac{\langle (|A|-|A^*|)x, x\rangle}{\langle (|A|+|A^*|)x, x\rangle} \right\}$.

References

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