

Constructive theory of orthogonal polynomials and new applications

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Constructive theory of orthogonal polynomials was developed in eighties in a series of papers by Walter Gautschi. It opened the door for extensive computational work on orthogonal polynomials and their applications not only in mathematics, but in other computational and applied sciences. Beside the basic procedures for numerical generation of coefficients in the three-term recurrence relation for orthogonal polynomials for arbitrary measures, in this lecture we present some details on the stability analysis of such algorithms, Christoffel modifications of the measure and corresponding algorithms, as well as available software. This theory enables the construction of many new classes of strongly non-classical orthogonal polynomials (very often with certain exotic weights), development of other types of orthogonality (s and σ -orthogonality, orthogonality on radial rays, Sobolev type of orthogonality, multiple orthogonality, etc.), applications in diverse areas of applied and numerical analysis (numerical integration, interpolation, integral equations, ...), approximation theory (moment-preserving spline approximation, ...), integration of fast oscillating functions, summation of slowly convergent series, etc. Particular attention will be paid to some of these issues.