

Computing asymptotic formulas for eigenvalues and eigenfunctions of a boundary value problem with retarded argument

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The paper is devoted to study the asymptotic behavior of eigenvalues and eigenfunctions of a boundary value problem which is generated by a differential equation with a retarded argument. The literature of the boundary value problems with retarded argument begins with the works of [1, 2, 3, 4, 5, 6]. Differential equations with retarded argument have many applications in the theory of automatic control, in the theory of self-oscillatory systems, in the study of problems connected with combustion in rocket engines, in a number of problems in economics, biophysics, and many other fields. Several physical applications of such problems can be found in [5].

Our problem differs from S. B. Norkin's [5] with the discontinuous coefficient in the differential equation and the discontinuities inside the interval in which we investigate the boundary value problem. These differences affect the expression of the equivalent integral expression for the solution of the boundary value problem which yields another difference in the expression of the characteristic equation. The characteristic equation of the boundary value problem plays a very important role while examining the properties of the eigenvalues and eigenfunctions.

Studies about the boundary value problems which is generated with a differential equation with retarded argument can not only be restricted to the investigation of the characteristics for the eigenvalues and eigenfunctions. For instance, in [7, 8], inverse Sturm-Liouville problems with a delay on finite interval are examined.

References

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