Finite element approximation of implicitly constituted fluid flow models

Endre Süli[1]

[1]Mathematical Institute, University of Oxford, U.K. endre.suli@maths.ox.ac.uk

Classical models describing the motion of Newtonian fluids, such as water, rely on the assumption that the Cauchy stress is a linear function of the symmetric part of the velocity gradient of the fluid. This assumption leads to the Navier-Stokes equations. It is known however that the framework of classical continuum mechanics, built upon an explicit constitutive equation for the Cauchy stress, is too narrow to describe inelastic behavior of solid-like materials or viscoelastic properties of materials. Our starting point in this work is therefore a generalization of the classical framework of continuum mechanics, called the *implicit constitutive theory*, which was proposed recently in a series of papers by Rajagopal. The underlying principle of the implicit constitutive theory in the context of viscous flows is the following: instead of demanding that the Cauchy stress is an explicit (and, in particular, linear) function of the symmetric part of the velocity gradient, one may allow a nonlinear, implicit and not necessarily continuous relationship between these quantities. The resulting general theory therefore admits non-Newtonian fluid flow models with implicit and possibly discontinuous power-law-like rheology.

We develop the analysis of finite element approximations of implicit power-law-like models for viscous incompressible fluids. The Cauchy stress and the symmetric part of the velocity gradient in the class of models under consideration are related by a, possibly multi-valued, maximal monotone graph. Using a variety of weak compactness techniques, including Chacon's biting lemma, we show that a subsequence of the sequence of finite element solutions converges to a weak solution of the problem as the discretization parameter, measuring the granularity of the finite element triangulation, tends to zero. A key new technical tool in our analysis is a finite element counterpart of the Acerbi-Fusco Lipschitz truncation of Sobolev functions.

The talk is based on a series of recent papers with Lars Diening (Bielefeld) and Christian Kreuzer (Dortmund), and ongoing research with Tabea Tscherpel (Oxford).