

On the symmetries of the 4-dimensional nilpotent Lie groups

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The symmetry of a tensor T is a 1-parameter group of diffeomorphisms of manifold (M, g) , leaving T invariant. Therefore, we consider a vector field X satisfying the condition $\mathcal{L}_X T = 0$, where \mathcal{L} denotes the Lie derivative. Examples of symmetries are isometries (for $T = g$ and X being a Killing vector field), but also homotheties, curvature collineations ($T = R$ where R is the curvature tensor), Ricci collineations (where $T = \rho$ is the Ricci tensor), Weyl collineations ($T = W$ being the Weyl conformal curvature tensor), etc.

We investigate symmetries of the four-dimensional nilpotent Lie groups, equipped with various left-invariant metrics of arbitrary signature. First, we give a full classification of left-invariant metrics on Lie groups $H_3 \times \mathbb{R}$ and G_4 and then we consider their geometry.

This is a part of an ongoing project with prof. Wafaa Batat, Ecole Nationale Polytechnique d'Oran, Algeria.