Some lower bounds of the energy of graphs

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Let G be a simple graph with the vertex set $V(G)$ and with the adjacency matrix $A(G)$. The energy $E(G)$ of G is defined to be the sum of the absolute values of all eigenvalues of $A(G)$. Also let n and m be number of edges and vertices of the graph respectively. A regular graph is a graph where each vertex has the same number of neighbours. Given a graph G, its line graph $L(G)$ is a graph such that each vertex of $L(G)$ represents an edge of G; and two vertices of $L(G)$ are adjacent if and only if their corresponding edges share a common endpoint in G. In this paper we show that for every regular graphs and also for every line graphs such that $\delta(G) \geq 3$ we have, $E(G) \geq \frac{2m}{n} + n - 1$. Also it was proved that for any bipartite graph G, $2\mu(G) \leqslant E(G)$ such that $\mu(G)$ is the matching number of G and equality holds if and only if G is the disjoint union of some complete bipartite graphs with perfect matchings and some isolated vertices. We generalize this result by showing that it holds for an arbitrary graph.

References

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