Face enumeration on matroid base polytope

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To a generalized permutohedron Q is associated the quasisymmetric function $F(Q)$, defined in [1], which enumerates positive integer lattice points lying in maximal cones of the normal fan of Q. In [2] is defined its weighted refinements $F_q(Q)$

$$F_q(Q) := \sum_{\omega \in \mathbf{Z}_+^n} q^{\mathrm{rk}_Q(\mathcal{F}_\omega)} x_{\omega_1} x_{\omega_2} \cdots x_{\omega_n},$$

where $\mathrm{rk}_Q(\mathcal{F}_\omega)$ is a rank function on the face poset of the standard permutohedron determined by Q. Particularly, the enumerator $F_q(Q)$ contains the information about the f-vector of the generalized permutohedron Q. We study the special case of matroid base polytopes $Q = P_M$ and calculate their f-vectors. We also show that the corresponding weighted quassisymmetric enumerator $F_q(P_M)$ coincides with a universal morphism of combinatorial Hopf algebras of matroids to quasisymmetric functions.

References

[1] L. Billera, N. Jia and V. Reiner, A quassisymetric function for matroids, European J. Combin. **30** (2009) 1727–1757.

[2] V. Grujić, M. Pešović and T. Stojadinović, Weighted quasisymmetric enumerator for generalized permutohedra, arXiv:1704.06715.