Face enumeration on matroid base polytope

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To a generalized permutohedron $Q$ is associated the quasisymmetric function $F(Q)$, defined in [1], which enumerates positive integer lattice points lying in maximal cones of the normal fan of $Q$. In [2] is defined its weighted refinements $F_q(Q)$

$$F_q(Q) := \sum_{\omega \in \mathbb{Z}^n_+} q^{\text{rk}_Q(F_\omega)} x_{\omega_1} x_{\omega_2} \cdots x_{\omega_n},$$

where $\text{rk}_Q(F_\omega)$ is a rank function on the face poset of the standard permutohedron determined by $Q$. Particularly, the enumerator $F_q(Q)$ contains the information about the $f$-vector of the generalized permutohedron $Q$. We study the special case of matroid base polytopes $Q = P_M$ and calculate their $f$-vectors. We also show that the corresponding weighted quasisymmetric enumerator $F_q(P_M)$ coincides with a universal morphism of combinatorial Hopf algebras of matroids to quasisymmetric functions.

References
