

Stochastic analysis of the predator–prey model with Allee effect on prey

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This paper presents the analysis of stochastic Rosenzweig–MacArthur predator–prey model with Allee effect on prey population of form

$$\begin{aligned}dx(t) &= x(t) \left[\frac{bx(t)}{A_1 + x(t)} - d_1 - \alpha x(t) - \frac{sy(t)}{1 + sh_1x(t)} \right] dt - \sigma_1 x(t) dw_1(t), \\dy(t) &= y(t) \left[\frac{c_1sx(t)}{1 + sh_1x(t)} - d_2 \right] dt - \sigma_2 y(t) dw_2(t),\end{aligned}$$

with initial value $x(0) = x_0$, $y(0) = y_0$. For this model, we first prove the existence and uniqueness of global positive solution by using the comparison theorem for stochastic differential equations. Then, we consider extinction of predator and prey population and conditions under which the extinction occurs. We also find the conditions for parameters of the model under which the solution of the system is globally attractive in mean. Finally, the numerical illustration with real life example is carried out to confirm our theoretical results. More precisely, we consider interaction between wolf and moose populations on Isle Royale to confirm our theoretical results.

References

- [1] W. C. Allee, *Animal Aggregations, a Study in General Sociology*, University of Chicago Press, Chicago, 1931.
- [2] I. Barbalat, *Systems d'equations differential d'oscillations nonlinearities*, Rev. Roumaine Math. Pures Appl. **4**(2) (1959), 267–270.
- [3] I. I. Gikhman and A. V. Skorokhod, *Stochastic Differential Equations*, Naukova Dumka, Kiev, 1968 (in Russian).

- [4] M. Jovanović and M. Krstić, The influence of time-dependent delay on behavior of stochastic population model with the Allee effect, *Appl. Math. Model.* **39** (2015), 733–746.
- [5] I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus*, 2nd edition, Springer-Verlag, Berlin, 1991.
- [6] P. E. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations*, Springer, Berlin, 1995.
- [7] M. Krstić and M. Jovanović, On stochastic population model with the Allee effect, *Math. Comput. Model.* **52** (2010), 370–379.
- [8] A. Liebhold, J. Bascompte, The Allee effect, stochastic dynamics and the eradication of alien species, *Ecol. Lett.* **6** (2003), 133–140.
- [9] X. Mao, *Stochastic Differential Equations and Applications*, 2nd edition, Horwood, Chichester, UK, 2007.
- [10] X. Mao, Stochastic version of the Lasalle theorem, *J. Differential Equations* **153** (1999), 175–195.
- [11] J. A. Vucetich and R. O. Peterson, The influence of prey consumption and demographic stochasticity on population growth rate of Isle Royale wolves *Canis lupus*, *Oikos*, **107** (2004), 309–320.
- [12] www.isleroyalewolf.org/sites/default/files/annual-report-pdfAnnualReport2015-forweb.pdf
- [13] www.sciencemag.org/news/2015/04/inbred-wolf-population-isle-royale-collapses
- [14] Q. Yang and D. Jiang, A note on asymptotic behaviors of stochastic population model with Allee effect, *Appl. Math. Model.* **35** (2011), 4611–4619.
- [15] B. Zimmermann, H. Sand, P. Wabakken, O. Liberg and H. P. Andreassen, Predator-dependent functional response in wolves: from food limitation to surplus killing, *Journal of Animal Ecology* **84** (2015), 102–112.
- [16] S. R. Zhou, Y. F. Liu and G. Wang, The stability of predator–prey systems subject to the Allee effects, *Theoretical Population Biology* **67** (2005), 23–31.
- [17] J. Zu and M. Mimura, The impact of Allee effect on a predator–prey system with Holling type II functional response, *Appl. Math. Comput.* **217** (2010), 3542–3556.