Structural examinations of graphs with smallest least eigenvalue

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Connected graphs of fixed order and size that minimize the least eigenvalue appear in several references published in a recent past. In all of them, the structure of such graphs is examined in details. For example, it is known that such a graph must be either bipartite or a join of two nested split graphs (not both totally disconnected). Moreover, if the graph is bipartite then it has a special structure, that is it must be a so-called double nested graph.

Our contribution to this problem refers to the non-bipartite case, where we identify the unique graph of fixed order n and size m that minimizes the least eigenvalue whenever its size satisfies $m = \lceil \frac{n}{2} \rceil \lfloor \frac{n}{2} \rfloor + a$, where a is a fixed constant in $\left[1, \left\lceil \frac{n}{2} \right\rceil - 1\right]$.

In the lecture, we present the entire background along with our results.