Error estimates for certain cubature rules

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We estimate the error of selected cubature formulae constructed by the product of Gauss quadrature rules. The cases of multiple and (hyper-)surface integrals over n-dimensional cube, simplex, sphere and ball are considered (see [16]). The error estimates are obtained as the absolute value of the difference between cubature formula constructed by the product of Gauss quadrature rules and cubature formula constructed by the product of corresponding Gauss-Kronrod or corresponding generalized averaged Gaussian quadrature rules. Generaziled averaged Gaussian quadrature rule $\widehat{G}_{2l+1}$ is $(2l+1)$-point quadrature formula. It has $2l+1$ nodes and the nodes of the corresponding Gauss rule G_l with l nodes form a subset, similar to the situation for the $(2l+1)$-point Gauss-Kronrod rule H_{2l+1} associated with G_l. The advantages of $\widehat{G}_{2l+1}$ are that it exists also when H_{2l+1} does not, and that the numerical construction of $\widehat{G}_{2l+1}$, based on recently proposed effective numerical procedure (see [24]), is simpler than the construction of H_{2l+1}.

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