Construction of the optimal set of quadrature rules for four integrals in the sense of Borges

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Borges in [1] considered a problem that arises in the evaluation of computer graphics illumination models. Starting with that problem, he examined the problem of numerically evaluating a set of $r \in \mathbb{N}$, $r \geq 2$, definite integrals of the form

$$\int_E f(x) w_j(x) \mathrm{d}x, \quad j = 1, 2, \dots, r,$$

where w_j , j = 1, 2, ..., r, are the weight functions.

We consider the problem of numerically evaluating a set of $r \in \mathbb{N}$, $r \geq 2$, definite integrals with the same integrand and over the same interval of integration, but with different weight functions, related to an arbitrary multi-index. Optimal set of quadrature rules for mentioned problem was studied in [1] and [2]. The stable numerical method for the construction of such optimal set of quadrature rules for two and three weight functions was given in [3].

In this paper we present numerical method for the construction of an optimal set of quadrature rules for four weight functions in the sense of Borges.

References

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