Improved estimates for the best constant in a Markov L_2-inequality

Geno Nikolov[1] and Rumen Uluchev[1]

[1]Department of Numerical Methods and Algorithms, Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski", geno@fmi.uni-sofia.bg, rumenu@fmi.uni-sofia.bg

Here we present new two-sided estimates for the best (i.e., the smallest possible) constant $c_n(\alpha)$ in the Markov inequality

$$\|p_n'\|_{w_\alpha} \le c_n(\alpha)\|p_n\|_{w_\alpha}, \quad p_n \in \mathcal{P}_n\,,$$

where $\mathcal{P}_n$ is the set of algebraic polynomials of degree at most n, $w_\alpha(x) := x^\alpha\, e^{-x}$, $\alpha > -1$, is the Laguerre weight function, and $\|\cdot\|_{w_\alpha}$ is the associated L_2-norm,

$$\|f\|_{w_\alpha} = \Big(\int_0^\infty w_\alpha(x)|f(x)|^2\, dx\Big)^{1/2}\,.$$

Our approach is based on the fact that $c_n^{-2}(\alpha)$ equals to the smallest zero of the $n^{\rm th}$ degree polynomial Q_n in a sequence of polynomials orthogonal with respect to a measure supported on $[0,\infty)$ and defined by an explicit three-term recurrence relation. We employ computer algebra to evaluate the seven lowest degree coefficients of Q_n and to obtain thereby bounds for $c_n(\alpha)$. This work is a continuation of a recent investigations [1], where estimates for $c_n(\alpha)$ were proven on the basis of the four lowest degree coefficients of Q_n.

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References

[1] G. Nikolov and A. Shadrin, On the L_2 Markov inequality with Laguerre weight, in: Progress in Approximation Theory and Applicable Complex Analysis' (N. K. Govil et al., eds.), Springer Optimization and Its Applications **117**, Springer Verlag, Berlin, 2017, pp. 1–17.