Heurističke metode optimizacije

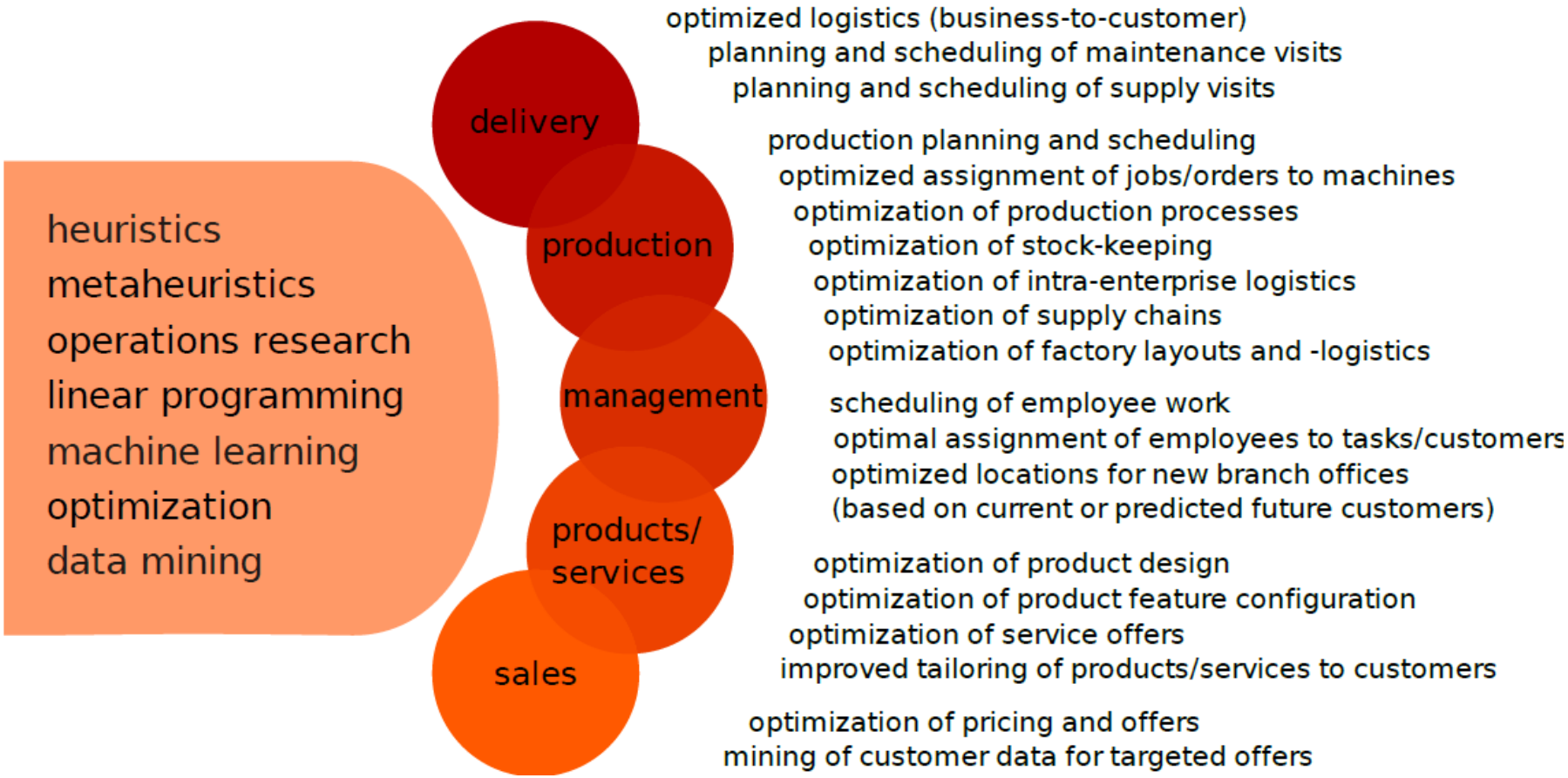
MAS Informatike – Nauka o podacima

Optimizacija

Optimizacija

- Cilj svake optimizacije je postepeno poboljšavanje performansi u pravcu idealnog optimuma.
- Optimizacija poboljšava rezultate tako što potpomaže odabir ulaza koji produkuju najbolje izlaze.
- Tehnike optimizacije svakodnevno se koriste u industrijskom planiranju, raspodeli resursa, pravljenju rasporeda, donošenja odluka, odabiru parametara modela itd.

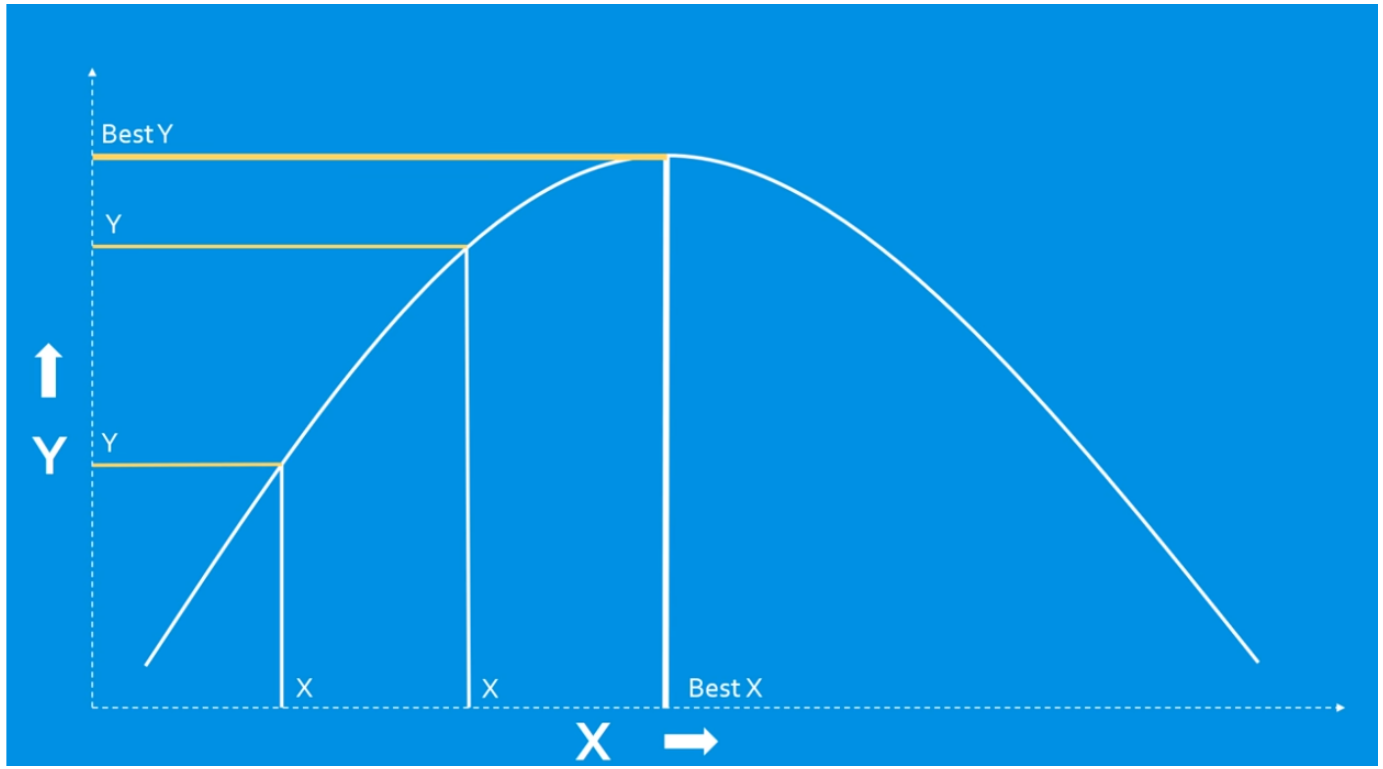
Primeri oblasti primene optimizacionih tehnika u poslovanju



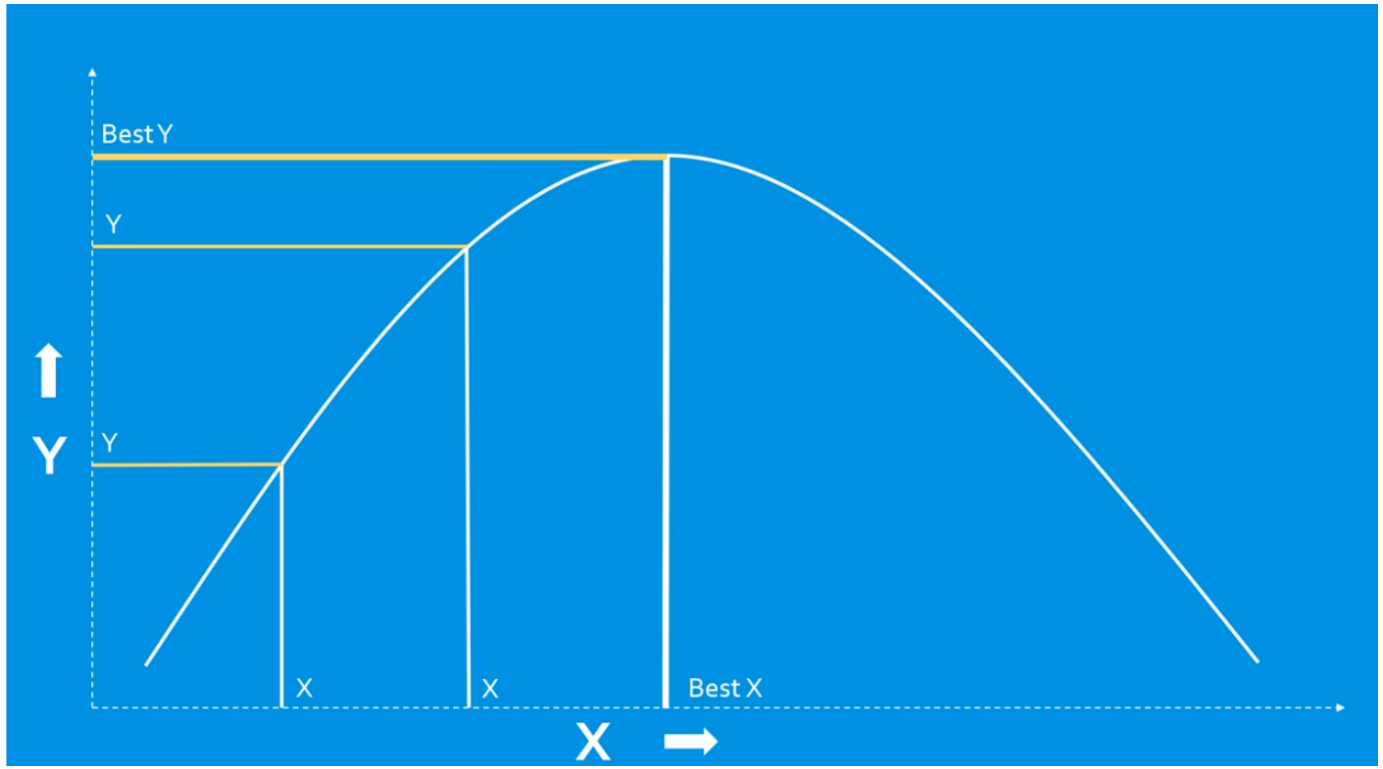
Optimizacija

- Optimizacija je proces rešavanja optimizacionog problema.
- Optimizacioni problem je situacija koja zahteva odabir jedne od mogućih alternativa u cilju dostizanja maksimalne dobiti.
- Optimizacioni problem je, sa matematičkog stanovišta, pronalaženje minimuma ili maksimuma neke funkcije – određivanje vrednosti x^* za koju funkcija f dostiže svoj optimum.

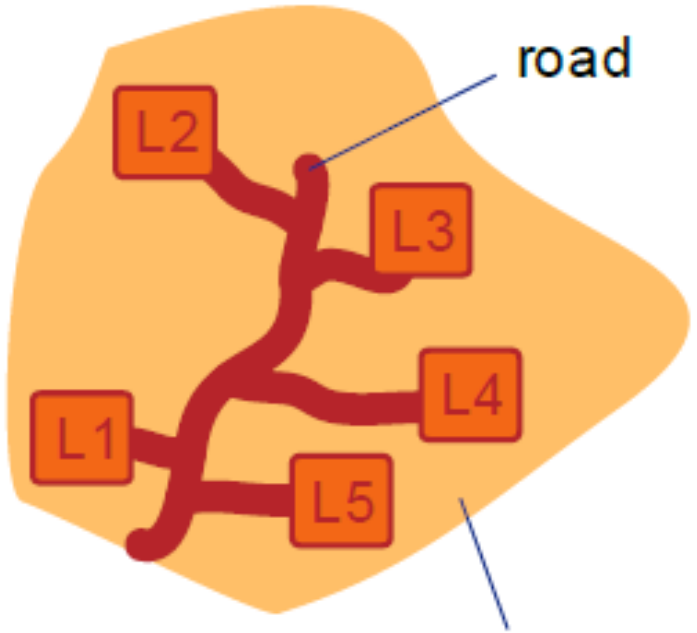
Primer



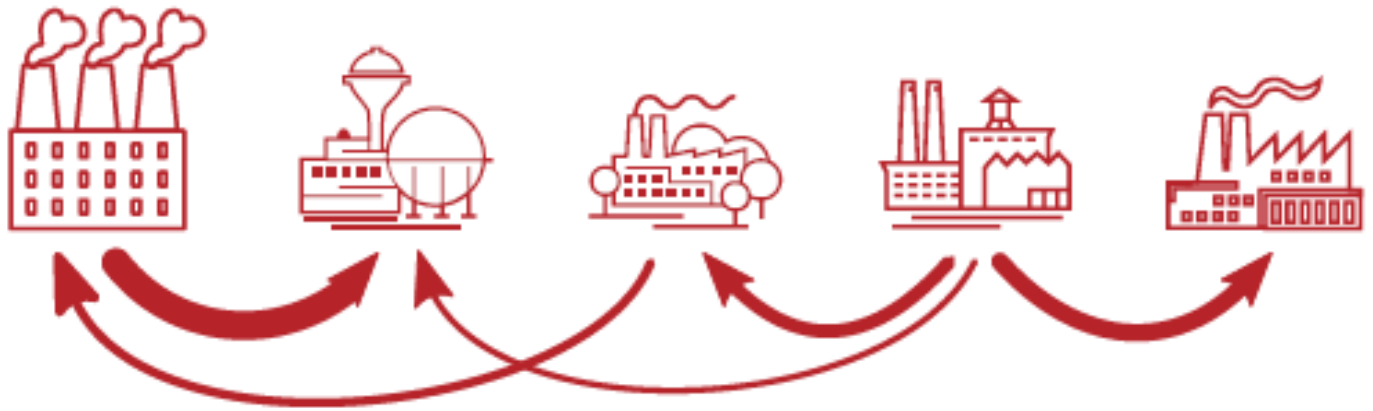
Primer



- Ovaj primer je jednostavan i lako se rešava pronalaženjem nule prvog izvoda funkcije.
- Funkcije mogu biti tako složene da je pronalaženje nula njihovog prvog izvoda jako komplikovano i dugotrajno.



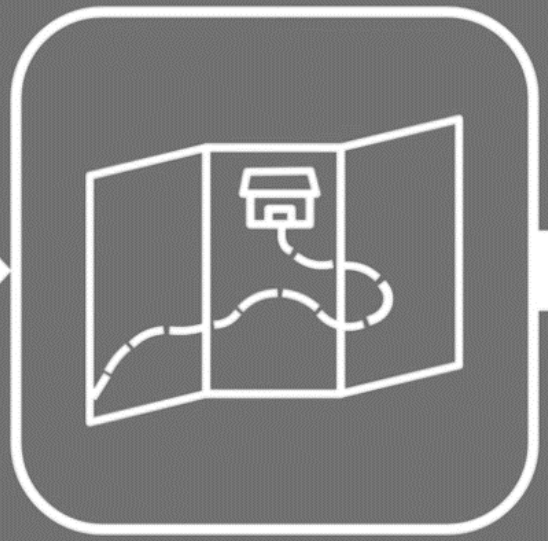
the land with 5 locations



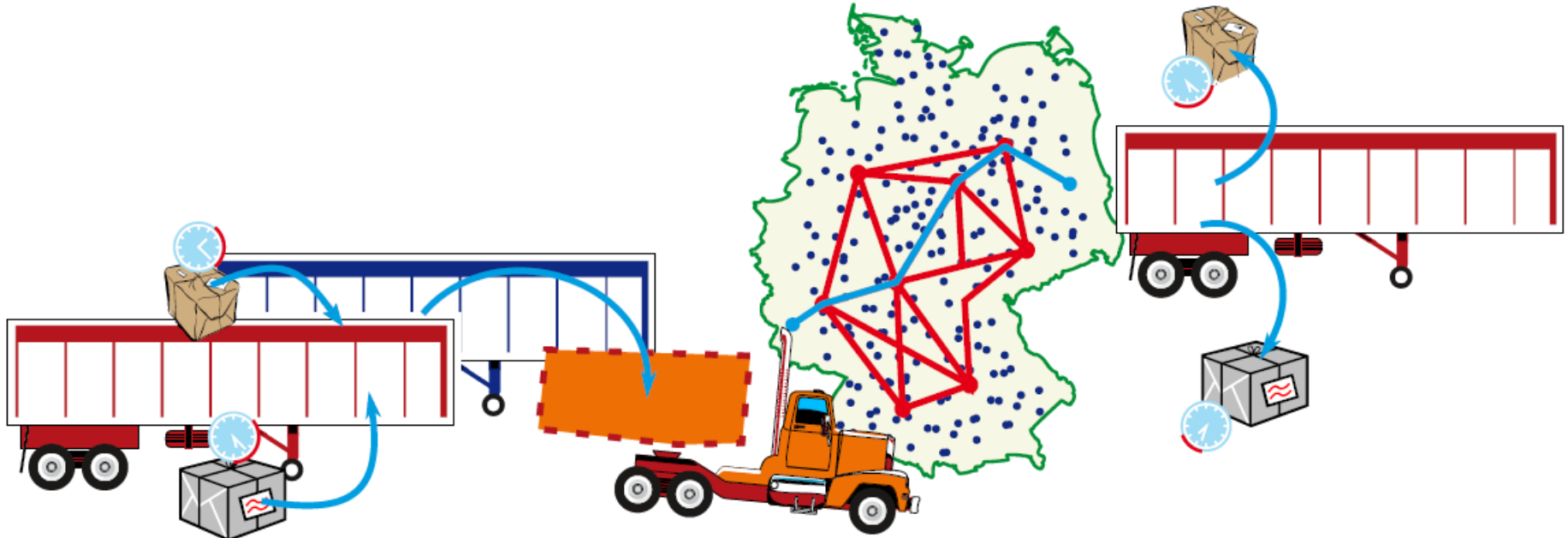
the 5 workshops which need to be assigned to the 5 locations and the different material flows between them

Warehouse Placement

Warehouse Location

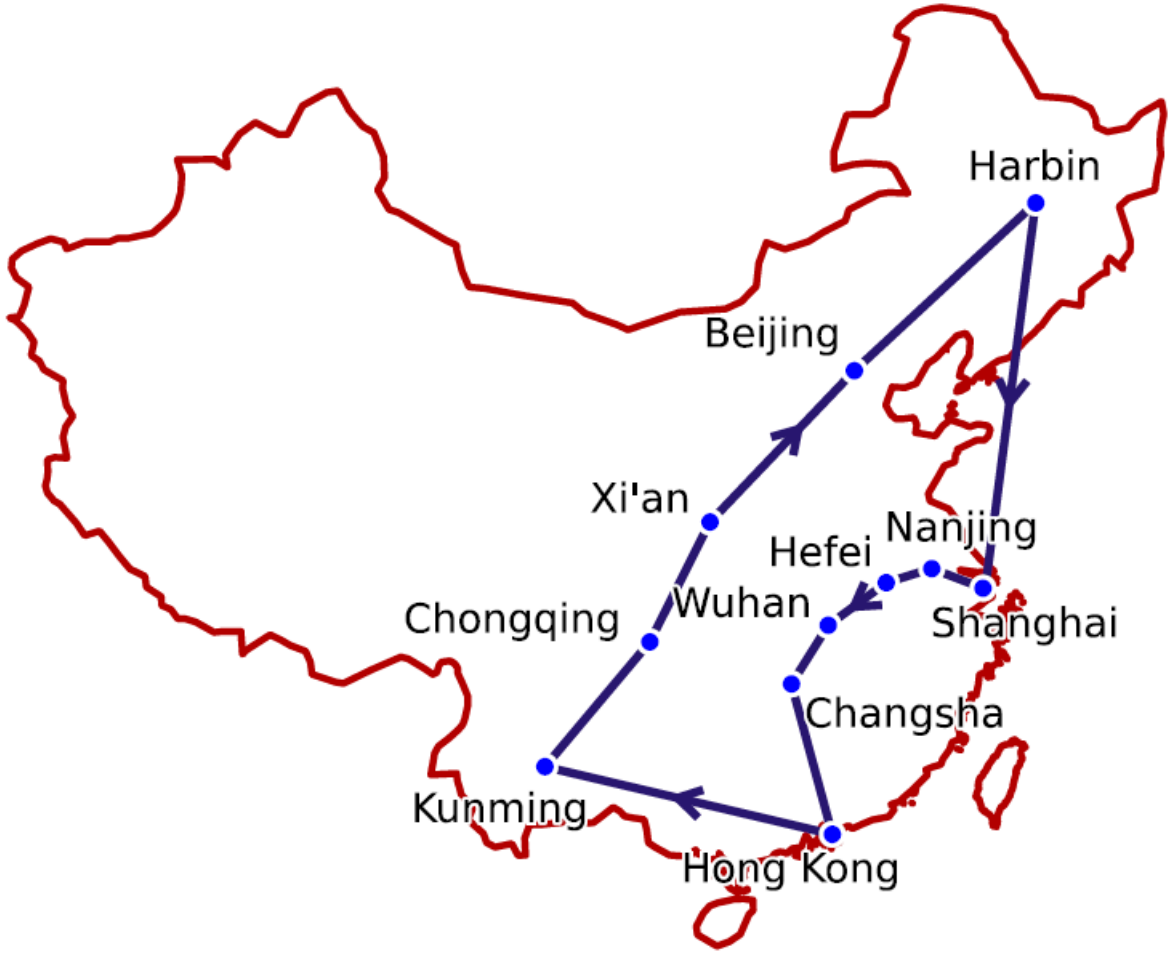


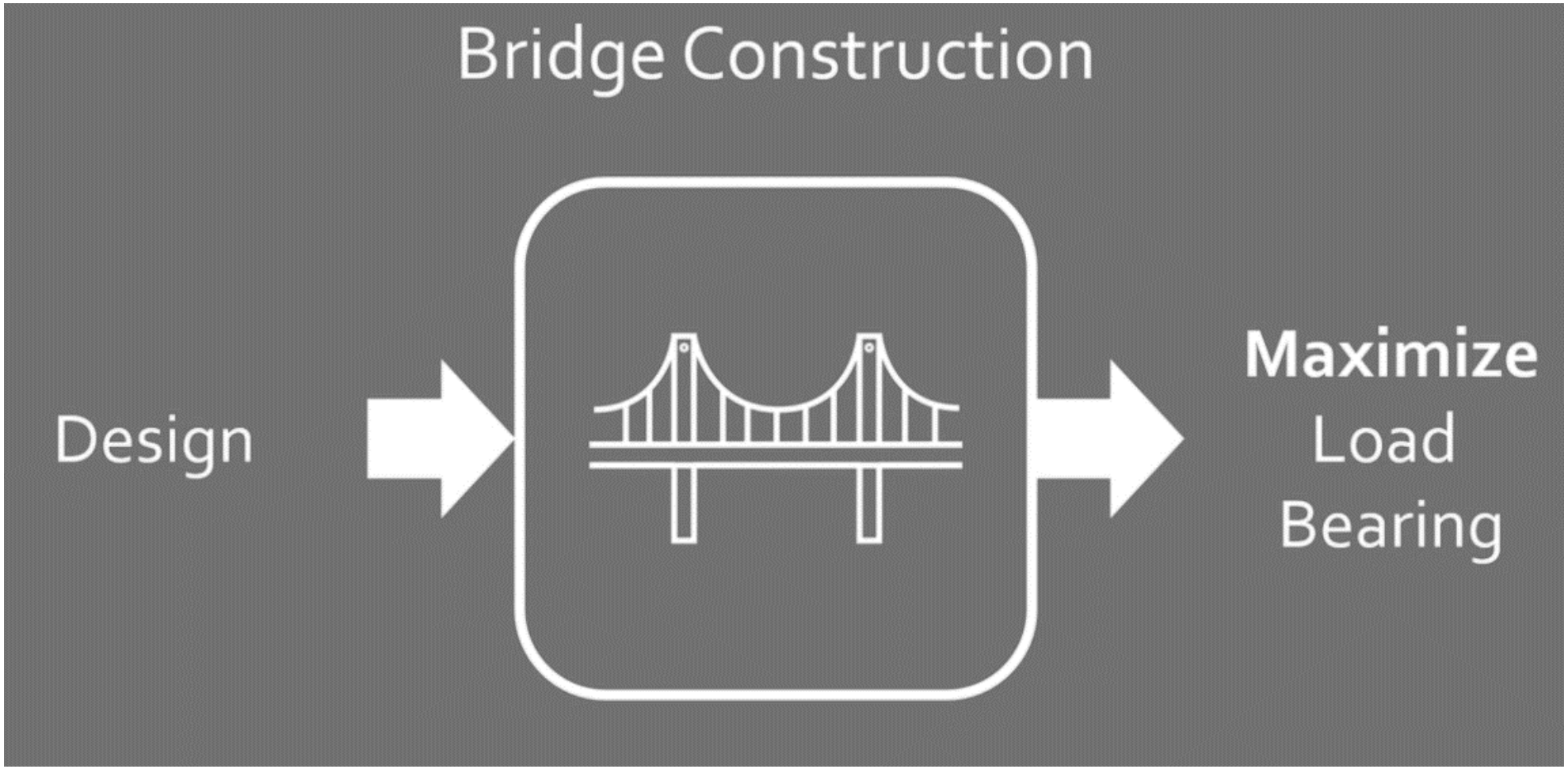
Minimize Shipment Time



Problem trgovačkog putnika (Traveling Salesman Problem (TSP))

- Cilj je pronaći najkraći put kroz n gradova.
- Mnogi drugi scenariji mogu se modelirati kao TSP:
  - Robotska ruka treba da buši nekoliko rupa na pločici

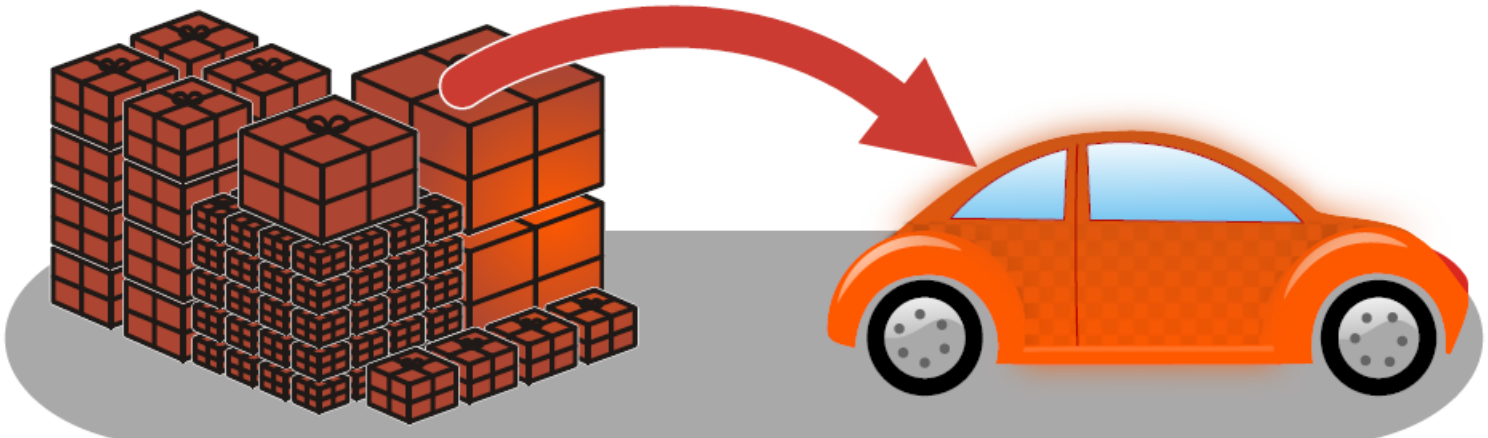


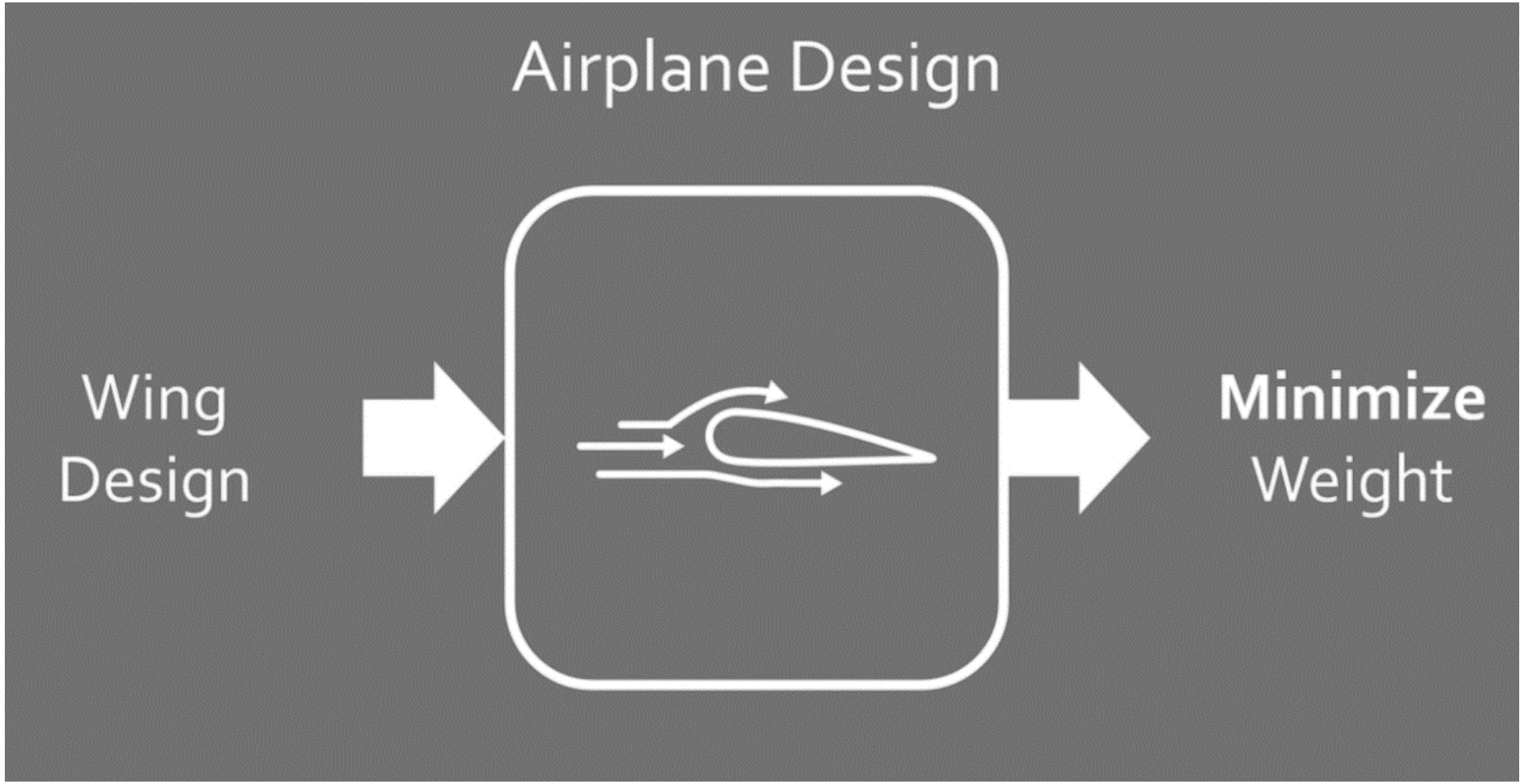

Bridge Construction
Design
Maximize
Load
Bearing

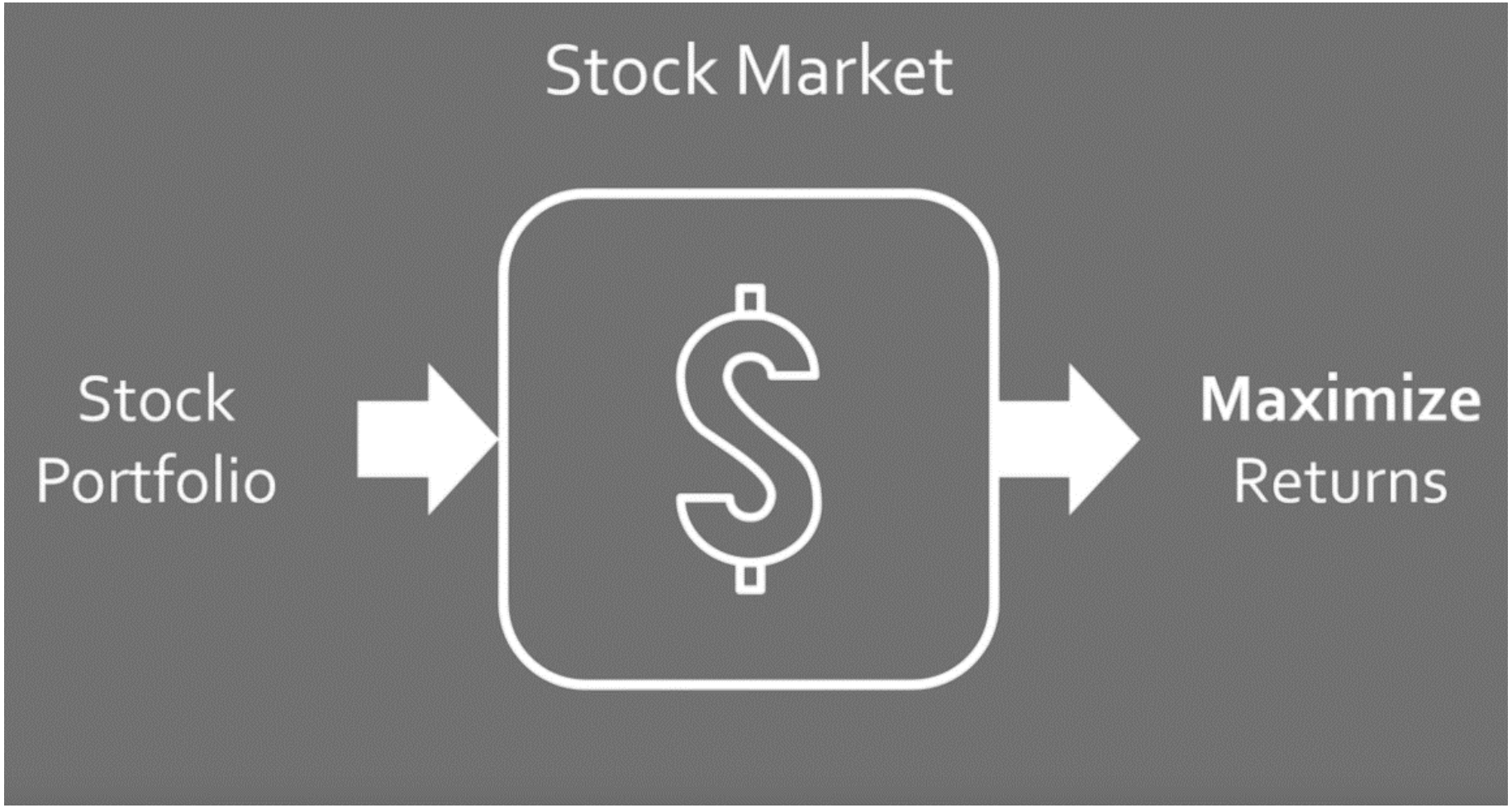
Problem pakovanja

- Cilj je da se predmeti što efikasnije spakuju u kontejnere, tj. na takav način da nam treba što manje kontejnera.

- Jednodimenzioni problem
- Dvodimenzioni problem
- Trodimenzioni problem

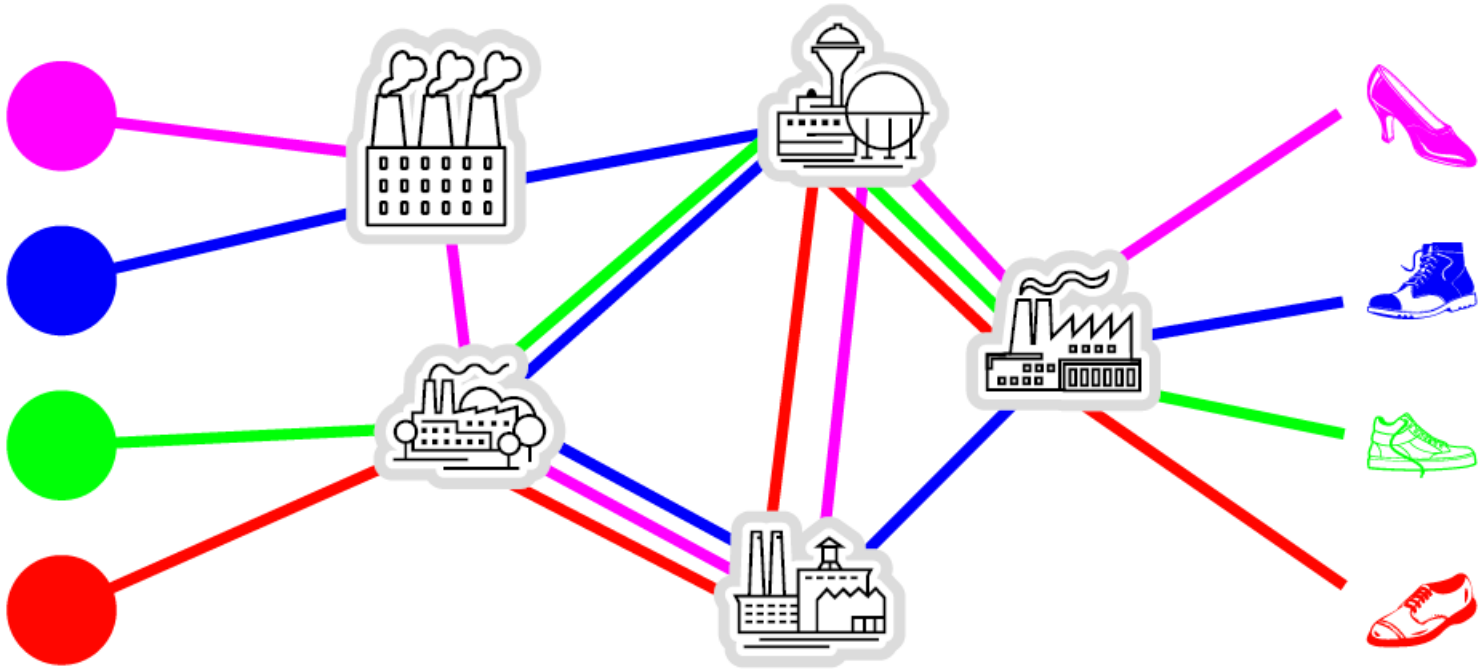


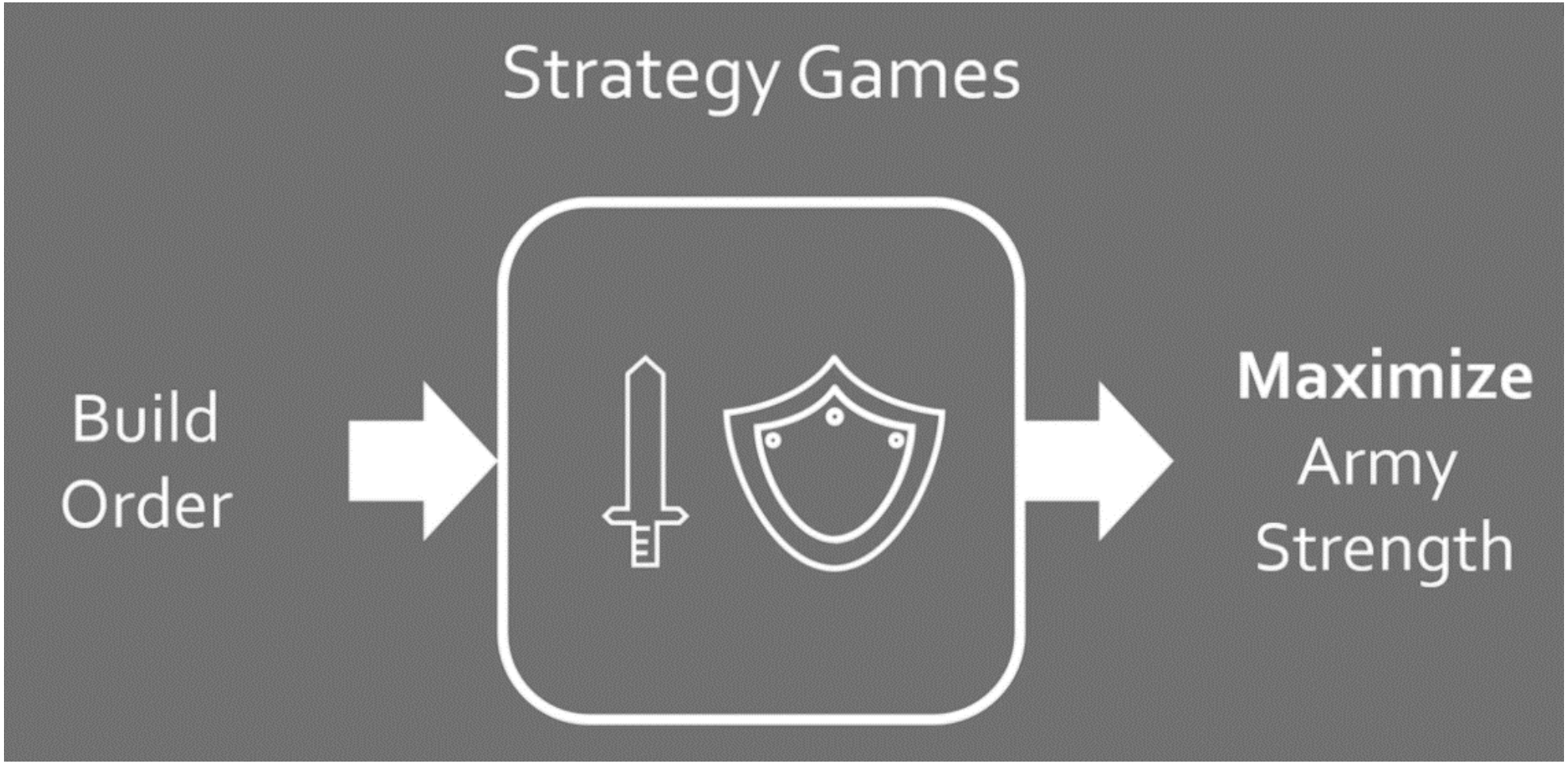

Airplane Design
Wing Design
Minimize Weight


Stock Market
Stock
Portfolio
Maximize
Returns

Problemi raspoređivanja (Job Shop Scheduling)

Dodela (“scheduling”) poslova (“jobs”) mašinama u fabrici („shop“) kako bi se optimizovale performanse proizvodnje.




Strategy Games
Build Order
Maximize Army Strength

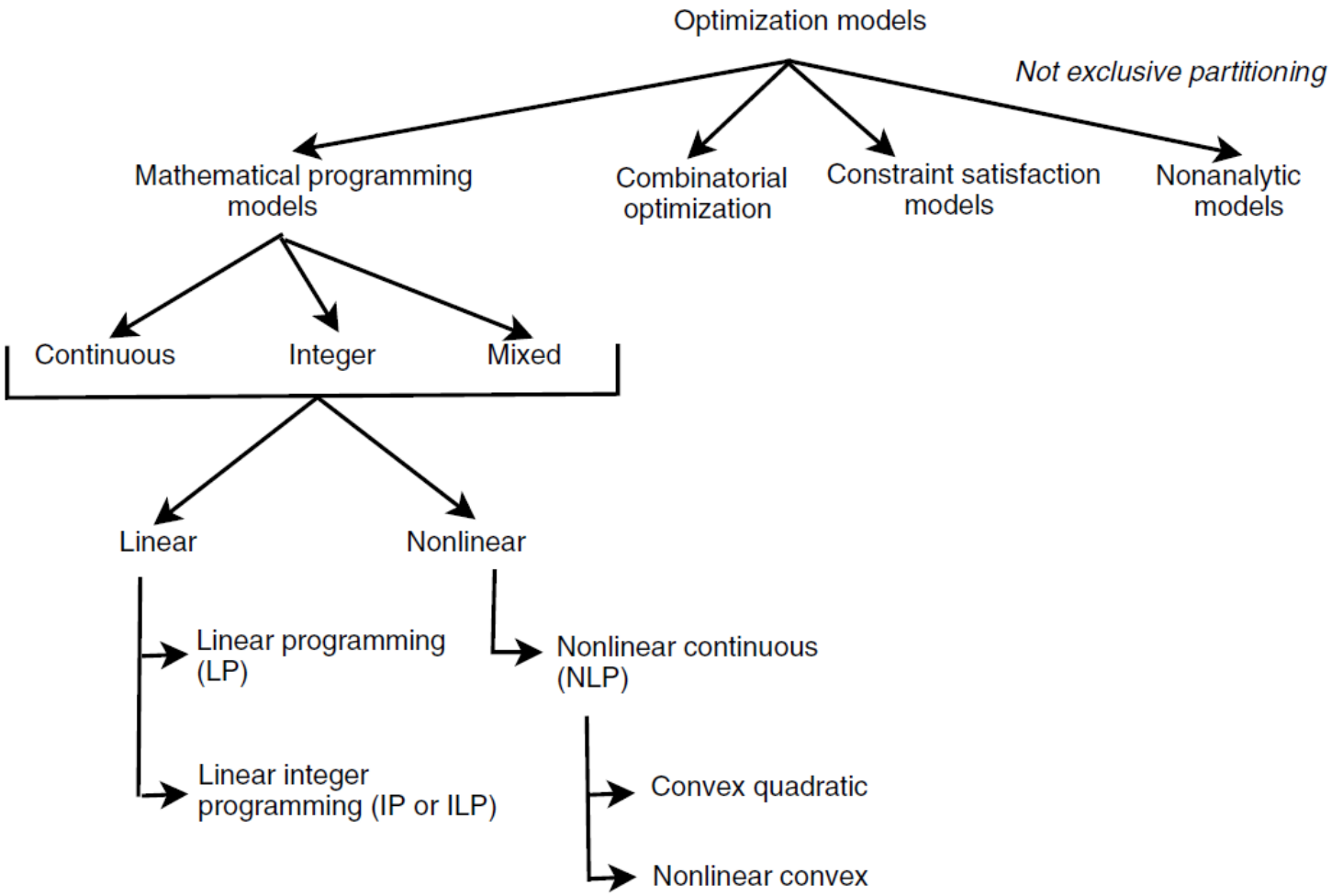
Primer jednostavnog problema optimizacije

In League of Legends, a player's Effective Health when defending against physical damage is given by $E = \frac{H(100+A)}{100}$, where H is health and A is armor.

(1) Health costs 2.5 gold per unit, and Armor costs 18 gold per unit. You have 3600 gold, and you need to optimize the effectiveness E of your health and armor to survive as long as possible against the enemy team's attacks. How much of each should you buy?

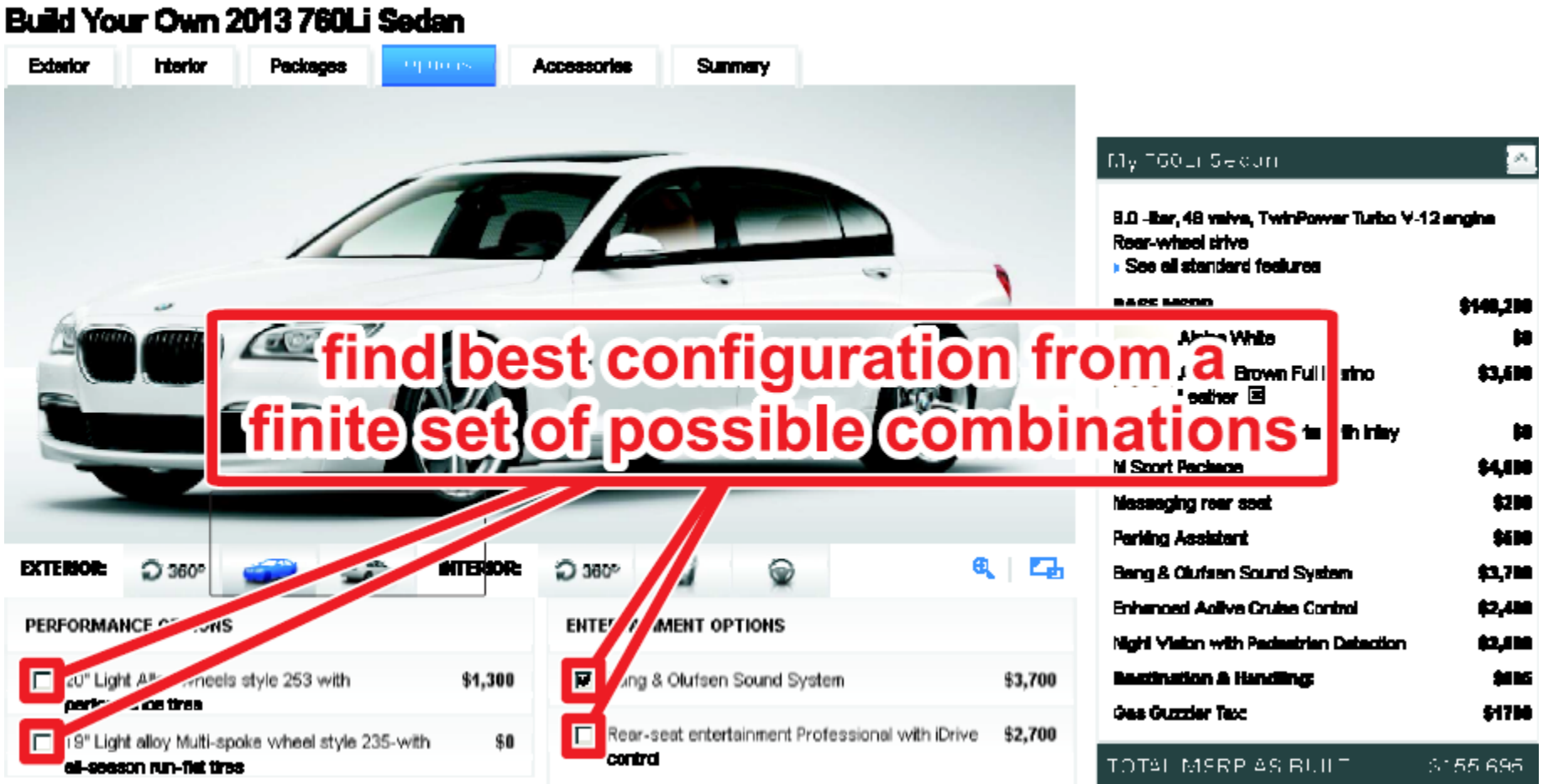
(2) Ten minutes into the game, you have 1080 health and 10 armor. You have only 720 gold to spend, and again Health costs 2.5 gold per unit while Armor costs 18 gold per unit. Again the goal is to maximize the effectiveness E. Notice that you don't want to maximize the effectiveness of what you purchase -- you want to maximize the effectiveness E of your *resulting* health and armor. How much of each should you buy?

Tipovi optimizacionih problema



Kombinatorni optimizacioni problemi

- Kombinatorni optimizacioni problemi su oni problemi koji imaju konačno mnogo diskretnih rešenja.



Kombinatorni optimizacioni problemi

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Find the visiting sequence of cities corresponding to the shortest round-trip tour.

Problemi koji imaju realne brojeve kao rešenja

Problemi koji imaju realne brojeve kao rešenja

Ingredients

125g butter, softened

100g light brown soft **sugar**

125g caster **sugar**

1 **egg**, lightly beaten

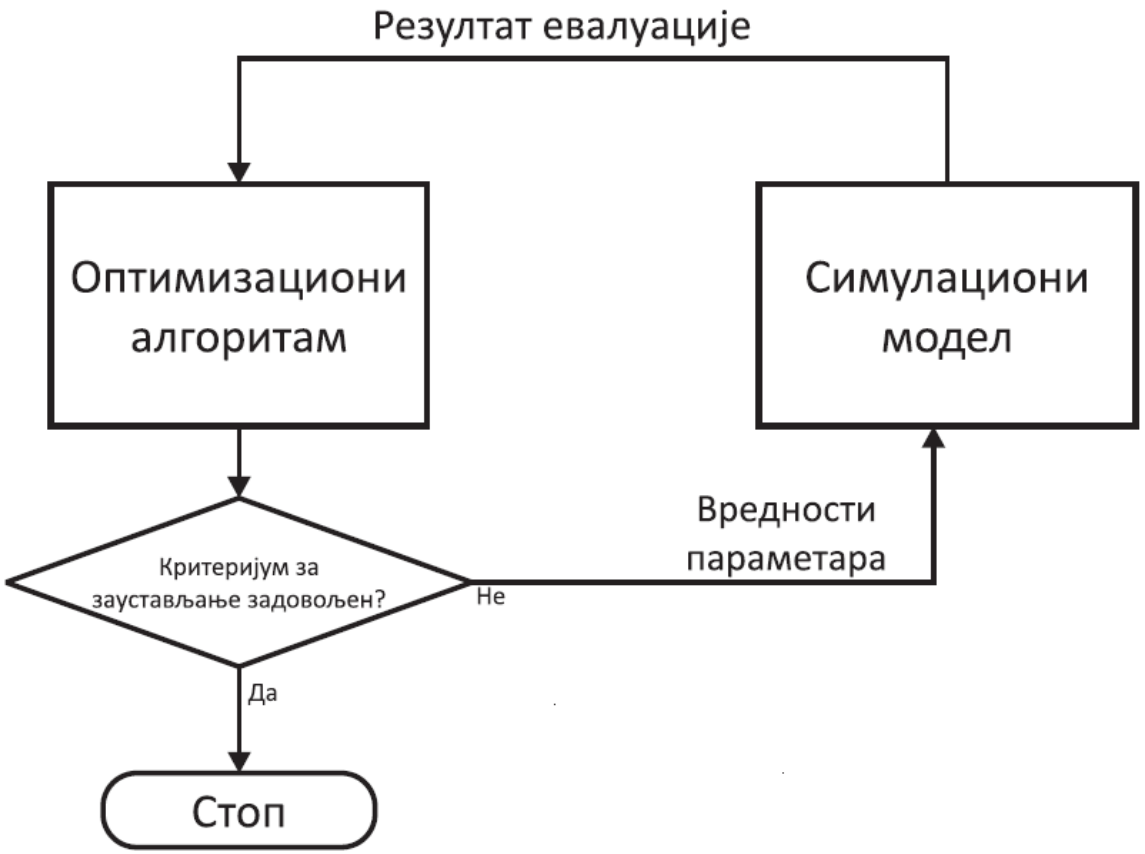
1 tsp **vanilla extract**

225g self-raising **flour**

Â½ tsp salt

200g chocolate chips

Optimizacija bazirana na simulaciji



Optimizacija - vokabular

Ciljna funkcija - Objective function

Variable odluke - Decision Variables

Ograničenja – Constraints

Optimizacioni problem

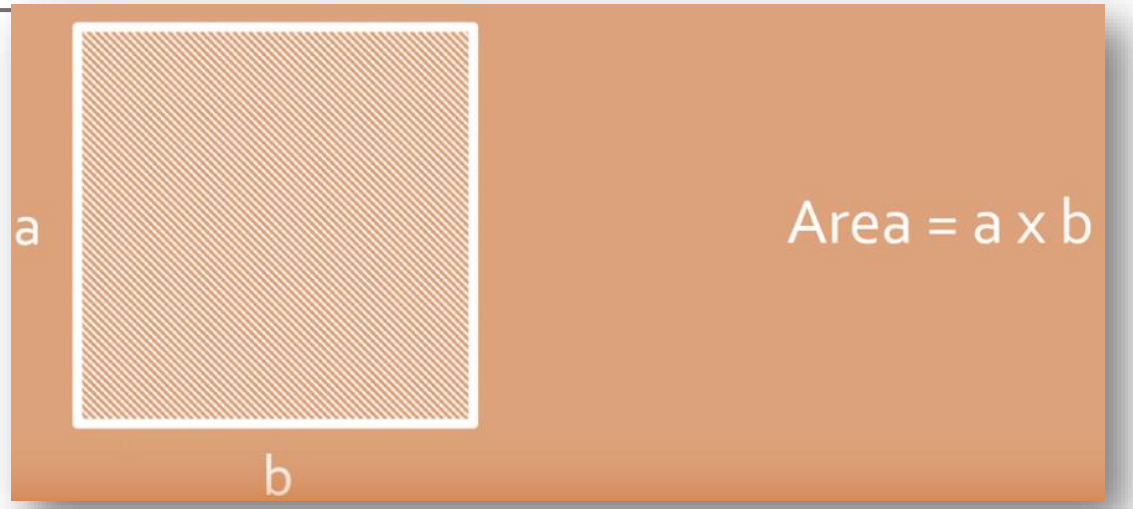
Optimization problem

Optimizacioni algoritam – Optimizational algorithm

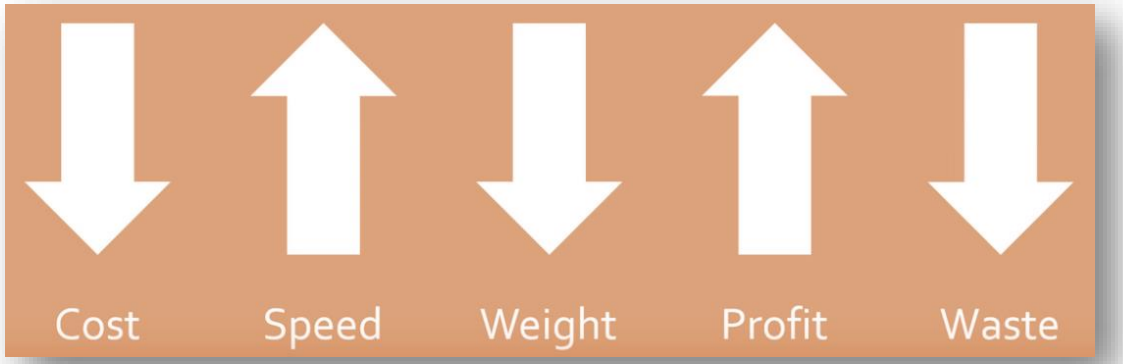
Ciljna funkcija

- Vrednost koju pokušavate da optimizujete.
  - Na primer, ako želite da od kartona koji imate napravite pravougaonik što veće površine, onda je površina pravougaonika ciljna funkcija.

$$min/max\ f(x)$$



- Glavni cilj svake optimizacije je poboljšanje vrednosti ciljne funkcije, tako da ona bude:
  - što veća (traženje maksimuma)
  - što manja (traženje minimuma)
  - što bliža nekoj unapred definisanoj vrednosti (traženje minimuma razlike)

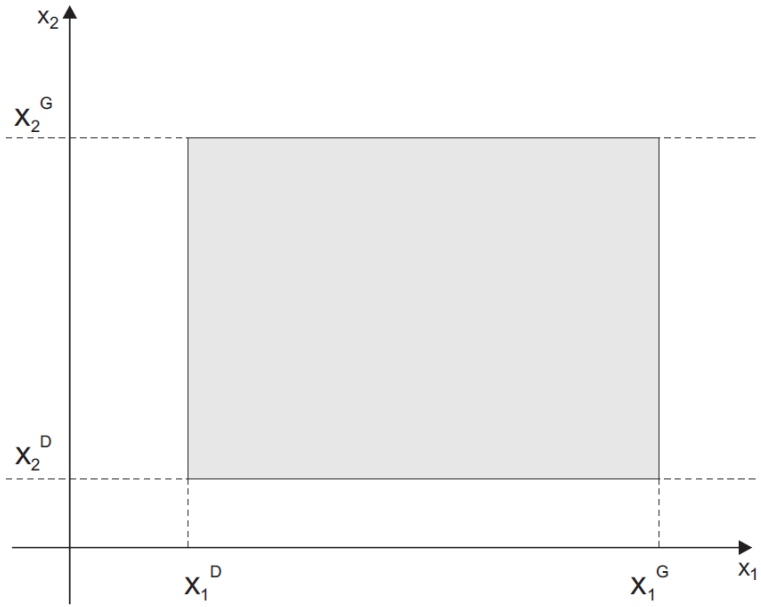


Variable odluke - Decision Variables

- Ulazi koje tokom procesa optimizacije možete da menjate.
- Promenom vrednosti promenljivih pokušavamo da poboljšamo vrednost ciljne funkcije.
  - U prethodnom primeru, promenom dimenzija a i b pokušavamo da maksimizujemo njegovu površinu.
- Za svaku promenljivu se definiše donja i gornja granica – opseg promenljive.
  - Na slici je dat primer prostora pretrage za slučaj kada imamo dve varijable odluke x_1 i x_2 i njihovi opsezi su (x_1^D, x_1^G) i (x_2^D, x_2^G)

$$min/max\ f(x_1, x_2)$$

$$x_1^D \leq x_1 \leq x_1^G$$
$$x_2^D \leq x_2 \leq x_2^G$$



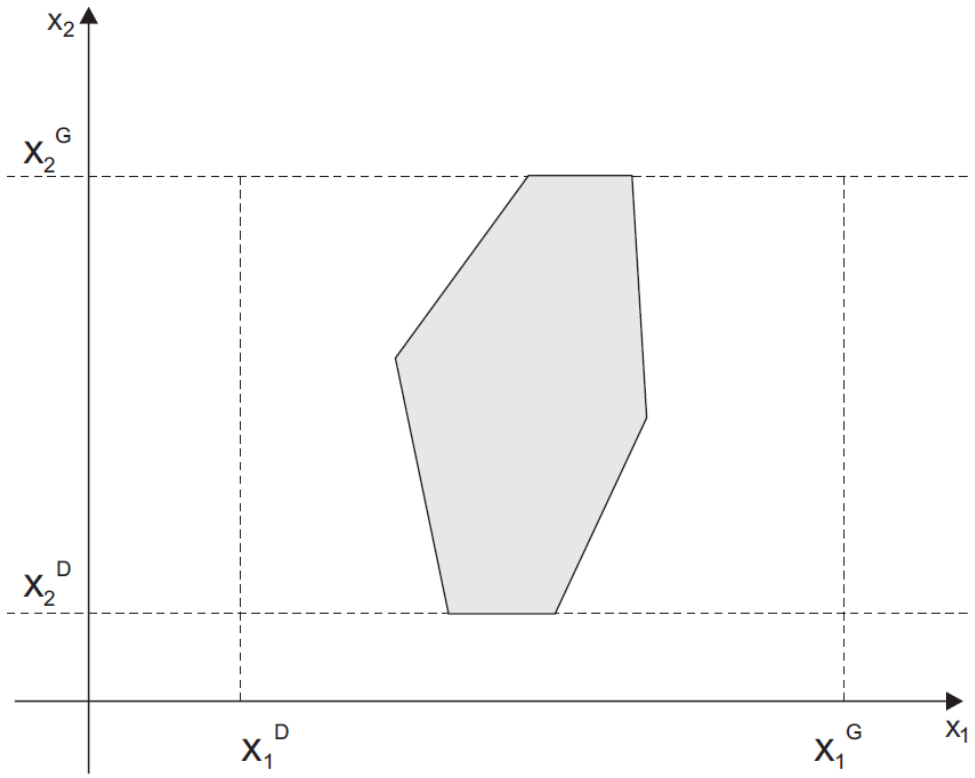
Ograničenja

- Odnosi između vrednosti varijabli odluke koji moraju da budu ispunjeni.
  - U primeru vezanom za *League of legends* ograničenje je da sredstva koja uložimo u kupovinu ne mogu biti veća od trenutno dostupnih.

$$2.5 \cdot H + 18 \cdot A \leq 3600$$

  - Na slici je dat primer prostora pretrage za slučaj kada imamo dve varijable odluke x_1 i x_2 i njihovi opsezi su (x_1^D, x_1^G) i (x_2^D, x_2^G) , a važe ograničenja:

$$h(x_1, x_2) \leq 0$$
$$g(x_1, x_2) \leq 0$$
$$\ldots$$



Optimizacioni problem

optimization problem = optimizacioni problem
optimization problem = problem optimizacije
optimization problem ≠ optimizacija problema

Matematička formulacija problema optimizacije:

$$\min / \max \ f\ (\boldsymbol{x}) \qquad \boldsymbol{x} = (x_1, x_2, \dots, x_n)$$
$$g_j(\boldsymbol{x}) \leq\ 0 \qquad j = 1,2, \dots, J$$
$$h_k\ (\boldsymbol{x}) =\ 0 \qquad k = 1,2, \dots, K$$
$$x_i^D \leq\ x_i \leq\ x_i^G \qquad i = 1,2, \dots, n$$

Jedna ciljna funkcija = Jednokriterijumska optimizacija

Primer

The Acme Bicycle Company produces two kinds of bicycles by hand: mountain bikes and street racers. Acme wishes to determine the rate at which each type of bicycle should be produced in order to maximize the profits on the sales of the bicycles. Acme assumes that it can sell all of the bicycles produced.

The physical data on the production process is available from the company engineer. A different team produces each kind of bicycle, and each team has a different maximum production rate: 2 mountain bikes per day and 3 racers per day, respectively. Producing a bicycle of either type requires the same amount of time on the metal finishing machine (a production bottleneck), and this machine can process at most a total of 4 bicycles per day, of either type. The company accountant estimates that mountain bikes are currently generating a profit of around $15 per bicycle, and racers a profit of around $10 per bicycle.