ЗАДАЦИ ЗА ДОМАЋИ

Диференцијалне једначине вишег реда

1. Решити једначине:

(1) $x = \frac{y''}{\sqrt{1+y'^2}}$,

(2) $y''' = \sqrt{1-x^2}$,

(3) $x - \sin y'' + 2y'' = 0$

(4) $x = e^{y''} + y'' = 0$,

(5) $y''^3 - 2y'' - x = 0$.

Решења:

(1) $y = -\frac{1}{2}\arcsin x - \frac{1}{2}x\sqrt{1-x^2} + c_1 x + c_2$,

(2) $x = \sin t, \quad y = -\frac{1}{8}\cos 2t + \frac{3}{16}t + \frac{7}{48}\sin 2t - \frac{1}{192}\sin 4t - \frac{1}{4}c_1\cos 2t + c_2\sin 2t + c_3$

(3) $x = \sin t - 2t, \quad y = \frac{3}{8}\sin 2t - \frac{1}{4}t\cos 2t - (2+t^2)\sin t + \frac{1}{2}t + \frac{2}{3}t^2 + c_1(\sin t - 2t) + c_2$,

(4) $x = e^t + t, \quad y = (\frac{t}{2} - \frac{3}{4})e^{2t} + (\frac{y^2}{2} - 1)e^t + \frac{t^3}{6} + c_1(e^t + t) + c_2$,

(5) $x = t^3 - 2t, \quad y = \frac{9}{28}t^7 - \frac{9}{10}t^5 + \frac{2}{3}t^3 + c_1(t^3 - 2t) + c_2$.

2. Решити једначине:

(1) $y''' = 2(y'' - 1)\cot x$,

(2) $y'' + 2y' = e^x y'^2$,

(3) $2xy'y'' = y'^2 - 1$,

(4) $y''^3 + xy'' = 2y'$,

(5) $2y'(y'' + 2) = xy''^2$,

(6) $y''^2 = 4(y' - 1)$,

(7) $xy'' = y' \ln \frac{y'}{x}$,

(8) $y' = xy'' + y''^2$,

(9) $xy'' = y' + x\sin \frac{y'}{x}$,

(10) $xy'' = y' + x(y'^2 + x^2)$.

Решења:

(1) $2y = c_1\cos 2x + (1 + 2c_1)x^2 + c_2 x + c_3$,

(2) $y = -e^{-x} - c_1 x + c_1 \ln|1 + c_1 e^x| + c_2, \quad y = c$,

(3) $9c_1^2(y - c_2)^2 = 4(c_1 x + 1)^3, \quad y = \pm x + c$,

(4) $x = c_1 t + 3t^2, \quad y = \frac{12}{5}t^5 + \frac{5}{4}c_1 t^4 + c_1^2\frac{t^3}{6} + c_2, \quad y = c$,

(5) $3c_1 y = (x - c_1)^3 + c_2, \quad y = c, \quad y = c - 2x^2$,

(6) $y = x + \frac{1}{3}(x + c_1)^3 + c + 2, \quad y_s = x + c$,

(7) $y = c_1 e^{\frac{x}{c_1}+1}(x - c_1) + c_2, \quad y = \frac{ex^2}{2} + c$,

(8) $y = \frac{c_1}{2}x^2 + c_1^2 x + c_2, \quad y_s = -\frac{x^3}{12} + c$,

(9) $y = (x^2 + c_1^2)\operatorname{arctg}\frac{x}{c_1} - c_1 x + c_2, \quad y = \frac{k\pi}{2}x^2 + c, k \in \mathbb{Z}$,

(10) $y = -\ln|\cos(\frac{x^2}{2} + c_1)| + c_2$.

3. Решити једначине:

(1) $y'' + y'^2 = 2e^{-y}$,
(2) $2yy'' - 3y'^2 - 4y^2 = 0$,
(3) $y''^2 - 2y'y''' + 1 = 0$,
(4) $yy''^2 = 1$,
(5) $y'''y'^2 = y''^3$,
(6) $(1 + y^2)yy'' = (3y^2 - 1)y'^2$,
(7) $2yy'' + y'^2 + y'^4 = 0$,
(8) $2yy'' = y'^2 + y^2$,
(9) $(y' + 2y)y'' = y'^2$,
(10) $y(1 - \ln y)y'' + (1 + \ln y)y'^2 = 0$.

Решења:

(1) $e^y + c_1 = (x + c_2)^2$,
(2) $y\cos^2(x + c_1) = c_2$,
(3) $12(c_1y - x) = c_1^2(x + c_2)^2 + c_3$,
(4) $(\pm 4\sqrt{y} + c_1)^{\frac{3}{2}} - 3c_1(\pm 4\sqrt{y} + c_1)^{\frac{1}{2}} = \pm 12x + c_2$,
(5) $x = \ln|t| + 2c_1t - c_2, \quad y = t + c_1t^2 + c_3, \quad y = c_1x + c_2$,
(6) $(1 + y^2)(c_1x + c_2) = 1, \quad y = c$,
(7) $(c_1y - 1)^{\frac{3}{2}} = \pm\frac{3c_1}{2}x + c_2$,
(8) $y + \frac{c_1}{2} + \sqrt{c_1y + y^2} = c_2e^{\pm x}, \quad y = 0$,
(9) $x = \pm\sqrt{1 + c_1y} - \ln|1 \pm \sqrt{1 + c_1y}| + c_2, \quad y = ce^{-x}, \quad y = c$,
(10) $\ln y = \frac{x+c_1}{x+c_2}, \quad y = c$.

4. Решити једначине:

(1) $xyy'' - xy'^2 - yy' = 0$,
(2) $x^2yy'' = (y - xy')^2$,
(3) $yy'' - 3y'^2 + 3yy' - y^2 = 0$,
(4) $(x^2 + 1)(y'^2yy'') = xyy'$,
(5) $yy'' = y'^2 + 15y^2\sqrt{x}$,
(6) $y'' + \frac{y'}{x} + \frac{y}{x^2} = \frac{y'^2}{y}$,
(7) $x^2(y'^2 - 2yy'') = y^2$,
(8) $xyy'' + (\frac{ax}{\sqrt{b^2-x^2}} - x)y'^2 - yy' = 0, \quad a, b \in \mathbb{R} \setminus \{0\}$,
(9) $ayy'' + by'^2 - \frac{yy'}{\sqrt{x^2+c^2}} = 0, \quad a, b, c \in \mathbb{R} \setminus \{0\}$.

Решења:

(1) $y = c_2e^{c_1x^2}$,
(2) $y = c_2xe^{-\frac{c_1}{x}}$,
(3) $(2e^x - c_1)y^2 = c_2e^{2x}$,
(4) $y = c_2(x + \sqrt{x^2 + 1})^{c_1}$,
(5) $y = c_2e^{4x^{\frac{5}{2}} + c_1x}$,
(6) $y = c_2|x|^{c_1 - \frac{1}{2}\ln|x|}$,
(7) $y = c_2x(c_1 + \ln|x|)^2, \quad y = cx$,
(8) $y = c_2e^{-\frac{\sqrt{b^2-x^2}}{a}}|a\sqrt{b^2 - x^2} + c_1|^{\frac{c_1}{a^2}}$,
(9) $y^{\frac{a+b}{a}} = c_1 + c_2(x + \sqrt{x^2 + c^2})^{\frac{1}{a}}(\sqrt{x^2 + c^2} - ax)$.

5. Решити једначине:

(1) $x^4y'' + (xy' - y)^3 = 0$,

(2) $x(x + y)y'' + xy'^2 + (x - y)y' - y = 0$,

(3) $x^2(x + y)y'' - (xy' - y)^2 = 0$,

(4) $\frac{y^2}{x^2} + y'^2 = 3xy'' + \frac{2yy'}{x}$,

(5) $x^3y'' = (y - xy')(y - xy' - x)$,

(6) $x^4(y'^2 - 2yy'') = 4x^3yy' + 1$,

(7) $4x^2y^3y'' = x^2 - y^4$,

(8) $xyy'' - 2xy'^2 + ayy' = 0$,

(9) $3x^2y''^2 - 2(3xy' + y)y'' + 4y'^2 = 0$

(10) $x^3y'' + 2xyy' - x^2y'^2 - y^2 = 0$,

(11) $x^4y'' - x^3y'^3 + 2x^2yy' - (3xy^2 + 2x^3)y' + 2x^2y + y^3 = 0$,

(12) $y'' = (2xy - \frac{5}{x})y' + 4y^2 - \frac{4y}{x^2}$,

(13) $xyy'' - 2xy'^2 + (y - 1)y' = 0$.

Решења:

(1) $y = x(c_1 + \arcsin\frac{c_2}{x})$,

(2) $(x + y)^2 = c_1x^2 + c_2, \quad y = 0$,

(3) $y = -x + xc_1e^{\frac{c_2}{x}}$,

(4) $y = x(c_2 - 3\ln|\frac{1}{x} - c_1|), \quad y = cx$,

(5) $y = -x\ln|c_1 + c_2\ln|x||, \quad y = cx$,

(6) $2c_2x^2y = (c_2x + c_1)^2 - 1, \quad xy = \pm 1$,

(7) $4c_1y^2 = x(4 + c_1^2(\ln|x| + c_2)^2)$,

(8) $y = \frac{1}{c_1 + c_2x^{1-a}}, a \neq 1, \quad y = \frac{1}{c_1 + c_2\ln|x|}, a = 1$,

(9) $y = c_1x^2 + c_1c_2x + c_2^2, \quad y = cx^{1\pm\frac{2}{\sqrt{3}}}$,

(10) $y = x\ln\frac{c_1x}{1+c_2x}, \quad y = cx$,

(11) $y = x(c_1 + \arcsin(c_2x))$,

(12) $x^2y = c_1\,\mathrm{tg}(c_1\ln|x| + c_2), \quad x^2y = c_1\frac{1+c_2|x|^{2c_1}}{1-c_2|x|^{2c_1}}, \quad x^2y(\ln|x| + c) + 1 = 0$,

(13) $2c_1y = \mathrm{tg}(c_1\ln|x| + c_2), \quad c_1y = \frac{c_2|x|^{c_1}-1}{c_2|x|^{c_1}+1}, \quad y = \frac{1}{2}\ln|x|, \quad y = c$.