ЗАДАЦИ ЗА ДОМАЋИ

Линеарне диференцијалне једначине са константним коефицијентима
Ојлерове диференцијалне једначине

1. Решити једначине:

(1) $y''' - y'' + y' - y = x^2 - x$,
(2) $y'' + 9y = x\cos x$,
(3) $y^{(5)} - 4y''' = 5$,
(4) $y'' - y = \sin^2 x$,
(5) $y'' + y = \sin^4 x$,
(6) $y'' - 4y' + 5y = (\sin x + 2\cos x)e^{2x}$,
(7) $y''' + 3y'' - 4y = xe^{-2x}$,
(8) $y''' - 3y'' + 3y' - y = 1 + e^x$,
(9) $y'' + y = \cos^2 x + \sin^2 \frac{x}{2}$,
(10) $y'' + 4y = e^x - 1 + 4\sin 2x + 2\cos^2 x$,
(11) $y'' - y = \frac{2e^x}{e^x - 1}$,
(12) $y'' + 4y = 2\operatorname{tg} x$,
(13) $y'' + ay = \cot ax, \quad a \in \mathbb{R}$,
(14) $y'' - y = 4\sqrt{x} + \frac{1}{x\sqrt{x}}$.

Решења:

(1) $y = c_1 e^x + c_2 \cos x + c_3 \sin x - (x^2 + 3x + 1)$,
(2) $y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{8} x\cos x + \frac{1}{32}\sin x$,
(3) $y = c_1 + c_2 x + c_3 x^2 + c_4 e^{2x} + c_5 e^{-2x} - \frac{5}{54}x^3$,
(4) $y = c_1 e^x + c_2 e^{-x} - \frac{1}{2} + \frac{1}{10}\cos 2x$,
(5) $y = c_1 \cos x + c_2 \sin x + \frac{3}{8} + \frac{1}{6}\cos 2x - \frac{1}{120}\cos 4x$,
(6) $y = (c_1 \cos x + c_2 \sin x)e^{2x} - \frac{1}{2}x(\cos x - 2\sin x)e^{2x}$,
(7) $y = c_1 e^x + (c_2 + c_3 x)e^{-2x} - \frac{1}{18}(x^3 + x^2)e^{-2x}$,
(8) $y = (c_1 + c_2 x + c_3 x^2)e^x - 1 + \frac{1}{6}x^3 e^x$,
(9) $y = c_1 \cos x + c_2 \sin x + 1 - \frac{1}{3}\cos 4x - \frac{1}{4}x\sin x$,
(10) $y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4}x(\sin 2x - 4\cos 2x) + \frac{1}{5}e^x$,
(11) $y = c_1 e^x + c_2 e^{-x} + 1 - xe^{-x} + 2\frac{e^x - e^{-x}}{2}\ln|1 - e^{-x}|$,
(12) $y = c_1 \sin 2x + c_2 \cos 2x + \sin 2x \ln|\cos 2x| - x\cos 2x$,
(13) $y = c_1 \cos ax + c_2 \sin ax + \frac{1}{a^2}\sin ax \ln|\frac{1 - \cos ax}{\sin ax}|$,
(14) $y = c_1 e^x + c_2 e^{-x} - 4\sqrt{x}$.

2. Решити једначине:

(1) $x^2 y'' + 4xy' + 2y = 2\ln^2 x + 12x$,
(2) $x^2 y'' - xy' - 3y = -\frac{16}{x}\ln x$,
(3) $x^2 y'' - xy' + 4y = \cos\ln x + x\sin\ln x$,
(4) $x^3 y''' + xy' - y = 3x^4$,
(5) $x^4 y^{(4)} + 6x^3 y''' + 4x^2 y'' - 2xy' - 4y = x^2 + 2\cos\ln x$,
(6) $x^4 y^{(4)} + 6x^3 y''' + 9x^2 y'' + 3xy' + y = x + 2\ln x + \ln^2 x$.

Решења:

(1) $y = \frac{c_1}{x} + \frac{c_2}{x^2} + \ln^2 x - 3\ln x + 2x + 7,$

(2) $y = \frac{1}{x}(c_1 + c_2 x^4 + \ln x + 2\ln^2 x),$

(3) $y = x(c_1 \cos(\sqrt{3}\ln x) + c_2 \sin(\sqrt{3}\ln x)) + \frac{1}{13}(3\cos\ln x - 2\sin\ln x) + \frac{1}{2}x\sin\ln x,$

(4) $y = c_1 x + c_2 x \ln|x| + c_3 x \ln^2|x| + \frac{1}{9}x^4,$

(5) $y = c_1 x^2 + \frac{c_2}{x^2} + c_3 \sin\ln x + c_4 \cos\ln x + \frac{x^2}{20}\ln x - \frac{1}{5}\ln x \sin\ln x,$

(6) $y = (c_1 + c_2 \ln x)\cos\ln x + (c_3 + c_4 \ln x)\sin\ln x + \frac{1}{4}x + \ln^2 x + 2\ln x - 4.$