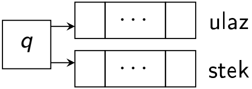
Formalni jezici, automati i jezički procesori
Formalni jezici i jezički procesori
Teorija automata i programski prevodioci

školska 2021/2022

Potisni automati- automati sa stekom



Дефиниција

Potisni automat je sedmorka $(Q, q_0, F, \Sigma, \Gamma, \$, \delta)$, *gde je*

- Q *konačan skup stanja,*
- $q_0 \in Q$ *početno stanje,*
- $F \subseteq Q$ *skup završnih stanja,*
- Σ *ulazni alfabet,*
- Γ *alfabet steka,*
- $\$ \in \Gamma \setminus \Sigma$ *početni simbol steka,*
- $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \to \mathcal{P}(Q \times (\Gamma \cup \{\varepsilon\}))$.

Potisni automati

- Početnu konfiguraciju potisnog automata za ulaz $w = u_1 \dots u_k \in \Sigma^*$ označavaćemo sa

$$q_0 \left[\begin{array}{l} u_1 \dots u_k \\ \$ \end{array} \right]$$

- Konfiguraciju čini trenutno stanje q, nepročitani deo ulaza $u_i \dots u_k$ i reč $s_1 \dots s_\ell$ iz Γ^* upisana na steku ($s_j \in \Gamma$):

$$(*) \qquad q \left[\begin{array}{l} u_i \dots u_k \\ s_1 \dots s_\ell \end{array} \right]$$

- Računski korak je promena konfiguracije u skladu sa definicijom funkcije δ.

Potisni automati

- Početnu konfiguraciju potisnog automata za ulaz $w = u_1 \ldots u_k \in \Sigma^*$ označavaćemo sa

$$q_0 \left[\begin{array}{l} u_1 \ldots u_k \\ \$ \end{array} \right]$$

- Konfiguraciju čini trenutno stanje q, nepročitani deo ulaza $u_i \ldots u_k$ i reč $s_1 \ldots s_\ell$ iz Γ^* upisana na steku ($s_j \in \Gamma$):

$$(*) \qquad q \left[\begin{array}{l} u_i \ldots u_k \\ s_1 \ldots s_\ell \end{array} \right]$$

- Računski korak je promena konfiguracije u skladu sa definicijom funkcije δ.

Potisni automati

- Početnu konfiguraciju potisnog automata za ulaz $w = u_1 \dots u_k \in \Sigma^*$ označavaćemo sa
$$q_0 \left[\begin{array}{l} u_1 \dots u_k \\ \$ \end{array} \right]$$
- Konfiguraciju čini trenutno stanje q, nepročitani deo ulaza $u_i \dots u_k$ i reč $s_1 \dots s_\ell$ iz Γ^* upisana na steku $(s_j \in \Gamma)$:
$$(*) \qquad q \left[\begin{array}{l} u_i \dots u_k \\ s_1 \dots s_\ell \end{array} \right]$$
- Računski korak je promena konfiguracije u skladu sa definicijom funkcije δ.

Potisni automat

Tekuća konfiguracija $(*)$ se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', s') \in \delta(q, u_i, s_1)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s' s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', s') \in \delta(q, \varepsilon, s_1)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s' s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', s') \in \delta(q, u_i, \varepsilon)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s' s_1 s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', s') \in \delta(q, \varepsilon, \varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s' s_1 s_2 \dots s_\ell \end{bmatrix}$$

Potisni automat

Tekuća konfiguracija $(*)$ se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \left[\begin{array}{l} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{array} \right] \xrightarrow{(q', s') \in \delta(q, u_i, s_1)} q' \left[\begin{array}{l} u_{i+1} \ldots u_k \\ s' s_2 \ldots s_\ell \end{array} \right]$$

$$q \left[\begin{array}{l} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{array} \right] \xrightarrow{(q', s') \in \delta(q, \varepsilon, s_1)} q' \left[\begin{array}{l} u_i u_{i+1} \ldots u_k \\ s' s_2 \ldots s_\ell \end{array} \right]$$

$$q \left[\begin{array}{l} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{array} \right] \xrightarrow{(q', s') \in \delta(q, u_i, \varepsilon)} q' \left[\begin{array}{l} u_{i+1} \ldots u_k \\ s' s_1 s_2 \ldots s_\ell \end{array} \right]$$

$$q \left[\begin{array}{l} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{array} \right] \xrightarrow{(q', s') \in \delta(q, \varepsilon, \varepsilon)} q' \left[\begin{array}{l} u_i u_{i+1} \ldots u_k \\ s' s_1 s_2 \ldots s_\ell \end{array} \right]$$

Potisni automat

Tekuća konfiguracija (∗) se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',s') \in \delta(q,u_i,s_1)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s' s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',s') \in \delta(q,\varepsilon,s_1)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s' s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',s') \in \delta(q,u_i,\varepsilon)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s' s_1 s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',s') \in \delta(q,\varepsilon,\varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s' s_1 s_2 \dots s_\ell \end{bmatrix}$$

Potisni automat

Tekuća konfiguracija $(*)$ se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', s') \in \delta(q, u_i, s_1)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s' s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', s') \in \delta(q, \varepsilon, s_1)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s' s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', s') \in \delta(q, u_i, \varepsilon)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s' s_1 s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', s') \in \delta(q, \varepsilon, \varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s' s_1 s_2 \dots s_\ell \end{bmatrix}$$

Potisni automat

Tekuća konfiguracija (∗) se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',s')\in\delta(q,u_i,s_1)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s' s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',s')\in\delta(q,\varepsilon,s_1)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s' s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',s')\in\delta(q,u_i,\varepsilon)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s' s_1 s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',s')\in\delta(q,\varepsilon,\varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s' s_1 s_2 \dots s_\ell \end{bmatrix}$$

Potisni automat

Tekuća konfiguracija $(*)$ se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix} \xrightarrow{(q',\varepsilon)\in\delta(q,u_i,s_1)} q' \begin{bmatrix} u_{i+1} \ldots u_k \\ s_2 \ldots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix} \xrightarrow{(q',\varepsilon)\in\delta(q,\varepsilon,s_1)} q' \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_2 \ldots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix} \xrightarrow{(q',\varepsilon)\in\delta(q,u_i,\varepsilon)} q' \begin{bmatrix} u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix} \xrightarrow{(q',\varepsilon)\in\delta(q,\varepsilon,\varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix}$$

Potisni automat

Tekuća konfiguracija $(*)$ se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, u_i, s_1)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, \varepsilon, s_1)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, u_i, \varepsilon)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, \varepsilon, \varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix}$$

Potisni automat

Tekuća konfiguracija $(*)$ se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, u_i, s_1)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, \varepsilon, s_1)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, u_i, \varepsilon)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, \varepsilon, \varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix}$$

Potisni automat

Tekuća konfiguracija $(*)$ se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',\varepsilon)\in\delta(q,u_i,s_1)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',\varepsilon)\in\delta(q,\varepsilon,s_1)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',\varepsilon)\in\delta(q,u_i,\varepsilon)} q' \begin{bmatrix} u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix} \xrightarrow{(q',\varepsilon)\in\delta(q,\varepsilon,\varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \dots u_k \\ s_1 s_2 \dots s_\ell \end{bmatrix}$$

Potisni automat

Tekuća konfiguracija (∗) se može promeniti izborom nekog elementa iz skupova $\delta(q, u_i, s_1)$, $\delta(q, \varepsilon, s_1)$, $\delta(q, u_i, \varepsilon)$ ili $\delta(q, \varepsilon, \varepsilon)$.

$$q \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, u_i, s_1)} q' \begin{bmatrix} u_{i+1} \ldots u_k \\ s_2 \ldots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, \varepsilon, s_1)} q' \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_2 \ldots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, u_i, \varepsilon)} q' \begin{bmatrix} u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix}$$

$$q \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix} \xrightarrow{(q', \varepsilon) \in \delta(q, \varepsilon, \varepsilon)} q' \begin{bmatrix} u_i u_{i+1} \ldots u_k \\ s_1 s_2 \ldots s_\ell \end{bmatrix}$$

Jezik automata

Дефиниција

Potisni automat $\mathbb{A}$ prihvata reč $w \in \Sigma^*$ (završnim stanjima i praznim stekom) ukoliko postoji niz računskih koraka koji početnu konfiguraciju

$$q_0 \begin{bmatrix} w \\ \$ \end{bmatrix}$$

transformiše u konfiguraciju

$$q \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix} \text{ za neko } q \in F.$$

Jezik koji automat $\mathbb{A}$ prihvata (svojim završnim stanjima i praznim stekom) je:

$$L(\mathbb{A}) = \left\{ w \in \Sigma^* \mid q_0 \begin{bmatrix} w \\ \$ \end{bmatrix} \vdash_{\mathbb{A}}^* q \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}, q \in F \right\}.$$

Jezik automata

Primer.

$$L = \{w\#w^R \mid w \in \{a,b\}^+\}$$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a, b, \#\}, \{a, b, \$\}, \$, \delta)$, pri čemu je δ:

$$\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\}\\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\}\\
\delta(q_0, \#, \varepsilon) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}$$

$$q_0 \begin{bmatrix} abb\#bba \\ \$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bb\#bba \\ a\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} b\#bba \\ ba\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} \#bba \\ bba\$ \end{bmatrix}$$

$$\vdash q_1 \begin{bmatrix} bba \\ bba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f$$

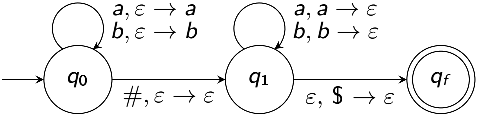
Jezik automata

Primer.

$$L = \{w\#w^R \mid w \in \{a,b\}^+\}$$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a, b, \#\}, \{a, b, \$\}, \$, \delta)$, pri čemu je δ:

$$\delta(q_0, a, \varepsilon) = \{(q_0, a)\}$$
$$\delta(q_0, b, \varepsilon) = \{(q_0, b)\}$$
$$\delta(q_0, \#, \varepsilon) = \{(q_1, \varepsilon)\}$$
$$\delta(q_1, a, a) = \{(q_1, \varepsilon)\}$$
$$\delta(q_1, b, b) = \{(q_1, \varepsilon)\}$$
$$\delta(q_1, \varepsilon, \$) = \{(q_f, \varepsilon)\}$$



$$q_0 \begin{bmatrix} abb\#bba \\ \$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bb\#bba \\ a\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} b\#bba \\ ba\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} \#bba \\ bba\$ \end{bmatrix}$$
$$\vdash q_1 \begin{bmatrix} bba \\ bba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f$$

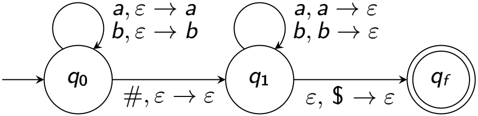
Jezik automata

Primer.

$$L = \{w\#w^R \mid w \in \{a,b\}^+\}$$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a, b, \#\}, \{a, b, \$\}, \$, \delta)$, pri čemu je δ:

$$\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\}\\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\}\\
\delta(q_0, \#, \varepsilon) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}$$



$$q_0\begin{bmatrix} abb\#bba \\ \$ \end{bmatrix} \vdash q_0\begin{bmatrix} bb\#bba \\ a\$ \end{bmatrix} \vdash q_0\begin{bmatrix} b\#bba \\ ba\$ \end{bmatrix} \vdash q_0\begin{bmatrix} \#bba \\ bba\$ \end{bmatrix}$$

$$\vdash q_1\begin{bmatrix} bba \\ bba\$ \end{bmatrix} \vdash q_1\begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1\begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1\begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f$$

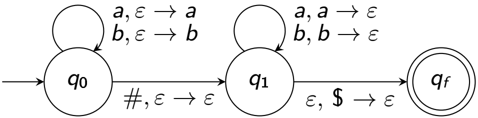
Jezik automata

Primer.

$L = \{w\#w^R \mid w \in \{a,b\}^+\}$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a, b, \#\}, \{a, b, \$\}, \$, \delta)$, pri čemu je δ:

$$
\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\}\\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\}\\
\delta(q_0, \#, \varepsilon) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}
$$



$$
q_0\begin{bmatrix} abb\#bba \\ \$ \end{bmatrix} \vdash q_0\begin{bmatrix} bb\#bba \\ a\$ \end{bmatrix} \vdash q_0\begin{bmatrix} b\#bba \\ ba\$ \end{bmatrix} \vdash q_0\begin{bmatrix} \#bba \\ bba\$ \end{bmatrix}
$$

$$
\vdash q_1\begin{bmatrix} bba \\ bba\$ \end{bmatrix} \vdash q_1\begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1\begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1\begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f
$$

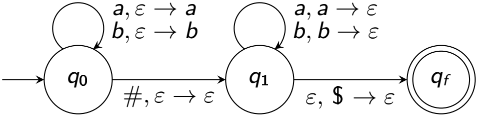
Jezik automata

Primer.

$L = \{w\#w^R \mid w \in \{a,b\}^+\}$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a, b, \#\}, \{a, b, \$\}, \$, \delta)$, pri čemu je δ:

$$
\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\}\\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\}\\
\delta(q_0, \#, \varepsilon) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}
$$



$$
q_0\begin{bmatrix} abb\#bba \\ \$ \end{bmatrix} \vdash q_0\begin{bmatrix} bb\#bba \\ a\$ \end{bmatrix} \vdash q_0\begin{bmatrix} b\#bba \\ ba\$ \end{bmatrix} \vdash q_0\begin{bmatrix} \#bba \\ bba\$ \end{bmatrix}
$$

$$
\vdash q_1\begin{bmatrix} bba \\ bba\$ \end{bmatrix} \vdash q_1\begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1\begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1\begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f
$$

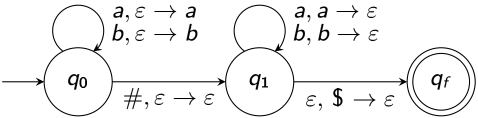
Jezik automata

Primer.

$L = \{w\#w^R \mid w \in \{a,b\}^+\}$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a, b, \#\}, \{a, b, \$\}, \$, \delta)$, pri čemu je δ:

$$
\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\}\\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\}\\
\delta(q_0, \#, \varepsilon) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}
$$



$$
q_0\begin{bmatrix} abb\#bba \\ \$ \end{bmatrix} \vdash q_0\begin{bmatrix} bb\#bba \\ a\$ \end{bmatrix} \vdash q_0\begin{bmatrix} b\#bba \\ ba\$ \end{bmatrix} \vdash q_0\begin{bmatrix} \#bba \\ bba\$ \end{bmatrix}
$$

$$
\vdash q_1\begin{bmatrix} bba \\ bba\$ \end{bmatrix} \vdash q_1\begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1\begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1\begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f
$$

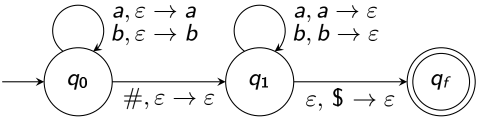
Jezik automata

Primer.

$L = \{w\#w^R \mid w \in \{a,b\}^+\}$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a, b, \#\}, \{a, b, \$\}, \$, \delta)$, pri čemu je δ:

$$\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\} \\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\} \\
\delta(q_0, \#, \varepsilon) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}$$



$$q_0 \begin{bmatrix} abb\#bba \\ \$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bb\#bba \\ a\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} b\#bba \\ ba\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} \#bba \\ bba\$ \end{bmatrix}$$

$$\vdash q_1 \begin{bmatrix} bba \\ bba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f$$

Jezik automata

Primer.

$L = \{ww^R \mid w \in \{a,b\}^+\}$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a,b\}, \{a,b,\$\}, \$, \delta)$, pri čemu je δ definisano sa:

$$\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\}\\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\}\\
\delta(q_0, \varepsilon, \varepsilon) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}$$

$$\begin{aligned}
q_0 \begin{bmatrix} abbbba \\ \$ \end{bmatrix} &\vdash q_0 \begin{bmatrix} bbbba \\ a\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bbba \\ ba\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bba \\ bba\$ \end{bmatrix}\\
&\vdash q_1 \begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}
\end{aligned}$$

Jezik automata

Primer.

$L = \{ww^R \mid w \in \{a,b\}^+\}$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a,b\}, \{a,b,\$\}, \$, \delta)$, pri čemu je δ definisano sa:

$$
\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\}\\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\}\\
\delta(q_0, \varepsilon, \varepsilon) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\}\\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}
$$

$$
q_0 \begin{bmatrix} abbbba \\ \$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bbbba \\ a\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bbba \\ ba\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bba \\ bba\$ \end{bmatrix}
$$

$$
\vdash q_1 \begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}
$$

Jezik automata

Primer.

$L = \{ww^R \mid w \in \{a, b\}^+\}$

$\mathbb{M} = (\{q_0, q_1, q_f\}, q_0, \{q_f\}, \{a, b\}, \{a, b, \$\}, \$, \delta)$, pri čemu je δ definisano sa:

$$\begin{aligned}
\delta(q_0, a, \varepsilon) &= \{(q_0, a)\} \\
\delta(q_0, b, \varepsilon) &= \{(q_0, b)\} \\
\delta(q_0, \varepsilon, \varepsilon) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, b, b) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, \varepsilon, \$) &= \{(q_f, \varepsilon)\}
\end{aligned}$$

$$\begin{aligned}
q_0 \begin{bmatrix} abbbba \\ \$ \end{bmatrix} &\vdash q_0 \begin{bmatrix} bbbba \\ a\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bbba \\ ba\$ \end{bmatrix} \vdash q_0 \begin{bmatrix} bba \\ bba\$ \end{bmatrix} \\
&\vdash q_1 \begin{bmatrix} ba \\ ba\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} a \\ a\$ \end{bmatrix} \vdash q_1 \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_f \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}
\end{aligned}$$

Jezik automata

Дефиниција

Jezik koji automat $\mathbb{A}$ prihvata svojim završnim stanjima je:

$$L_f(\mathbb{A}) = \left\{ w \in \Sigma^* \mid q_0 \begin{bmatrix} w \\ \$ \end{bmatrix} \vdash^*_{\mathbb{A}} q \begin{bmatrix} \varepsilon \\ \sigma \end{bmatrix}, q \in F, \sigma \in \Gamma^* \right\}$$

Jezik koji prihvata automat $\mathbb{A}$ praznim stekom je:

$$L_e(\mathbb{A}) = \left\{ w \in \Sigma^* \mid q_0 \begin{bmatrix} w \\ \$ \end{bmatrix} \vdash^*_{\mathbb{A}} q \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}, q \in Q \right\}.$$

Ukoliko za neki potisni automat $\mathbb{A} = (Q, q_0, F, \Sigma, \Gamma, \$, \delta)$, posmatramo jezik $L_e(\mathbb{A})$, onda uzimamo da je $F = \emptyset$, jer završna stanja nisu ni od kakvog značaja.

Jezik automata

Дефиниција

Jezik koji automat $\mathbb{A}$ prihvata svojim završnim stanjima je:

$$L_f(\mathbb{A}) = \left\{ w \in \Sigma^* \mid q_0 \begin{bmatrix} w \\ \$ \end{bmatrix} \vdash_{\mathbb{A}}^{*} q \begin{bmatrix} \varepsilon \\ \sigma \end{bmatrix}, q \in F, \sigma \in \Gamma^* \right\}$$

Jezik koji prihvata automat $\mathbb{A}$ praznim stekom je:

$$L_e(\mathbb{A}) = \left\{ w \in \Sigma^* \mid q_0 \begin{bmatrix} w \\ \$ \end{bmatrix} \vdash_{\mathbb{A}}^{*} q \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}, q \in Q \right\}.$$

Ukoliko za neki potisni automat $\mathbb{A} = (Q, q_0, F, \Sigma, \Gamma, \$, \delta)$, posmatramo jezik $L_e(\mathbb{A})$, onda uzimamo da je $F = \emptyset$, jer završna stanja nisu ni od kakvog značaja.

Jezik automata

Označimo sa $\mathcal{L}_{\mathrm{fe}}(\Sigma)$, $\mathcal{L}_{\mathrm{e}}(\Sigma)$ i $\mathcal{L}_{\mathrm{f}}(\Sigma)$ skupove jezika nad Σ za koje postoji potisni automat kojih ih redom prihvata

- praznim stekom i završnim stanjima,
- praznim stekom,
- završnim stanjima.

$$\begin{array}{lclcl} \mathcal{L}_{\mathrm{fe}}(\Sigma) & \searrow & & \nearrow & L = L_{\mathrm{fe}}(\mathbb{A}). \\ \mathcal{L}_{\mathrm{e}}(\Sigma) & \rightarrow & \text{skup jezika } L \subseteq \Sigma^* \text{ za koje postoji p.a. } \mathbb{A} \text{ t.d.} & \rightarrow & L = L_{\mathrm{e}}(\mathbb{A}). \\ \mathcal{L}_{\mathrm{f}}(\Sigma) & \nearrow & & \searrow & L = L_{\mathrm{f}}(\mathbb{A}). \end{array}$$

Теорема

$$\mathcal{L}_{\mathrm{fe}}(\Sigma) = \mathcal{L}_{\mathrm{e}}(\Sigma) = \mathcal{L}_{\mathrm{f}}(\Sigma)$$

Jezik automata

Označimo sa $\mathcal{L}_{\mathrm{fe}}(\Sigma)$, $\mathcal{L}_{\mathrm{e}}(\Sigma)$ i $\mathcal{L}_{\mathrm{f}}(\Sigma)$ skupove jezika nad Σ za koje postoji potisni automat kojih ih redom prihvata

- praznim stekom i završnim stanjima,
- praznim stekom,
- završnim stanjima.

$$\begin{array}{lcccl} \mathcal{L}_{\mathrm{fe}}(\Sigma) & \searrow & & \nearrow & L = L_{\mathrm{fe}}(\mathbb{A}). \\ \mathcal{L}_{\mathrm{e}}(\Sigma) & \rightarrow & \text{skup jezika } L \subseteq \Sigma^* \text{ za koje postoji p.a. } \mathbb{A} \text{ t.d.} & \rightarrow & L = L_{\mathrm{e}}(\mathbb{A}). \\ \mathcal{L}_{\mathrm{f}}(\Sigma) & \nearrow & & \searrow & L = L_{\mathrm{f}}(\mathbb{A}). \end{array}$$

Теорема

$$\mathcal{L}_{\mathrm{fe}}(\Sigma) = \mathcal{L}_{\mathrm{e}}(\Sigma) = \mathcal{L}_{\mathrm{f}}(\Sigma)$$

Jezik automata

Dokaz.

Inkluzije $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{e}}(\Sigma)$ i $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{f}}(\Sigma)$ su očigledne.

Dokažimo $\mathcal{L}_{\mathrm{e}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{fe}}(\Sigma)$.

Neka $L \in \mathcal{L}_{\mathrm{e}}(\Sigma)$. Tada postoji potisni automat $\mathbb{A} = (Q, q_0, \emptyset, \Sigma, \Gamma, \$, \delta)$ takav da je $L = L_{\mathrm{e}}(\mathbb{A})$. Nije teško konstruisati potisni automat $\mathbb{A}'$ koji:

- simulira $\mathbb{A}$ i
- kada $\mathbb{A}$ isprazni svoj stek, $\mathbb{A}'$ ulazi u završno stanje.

$\mathbb{A}' = (Q \cup \{q_0', q_{\mathrm{f}}'\}, q_0', \{q_{\mathrm{f}}'\}, \Sigma, \Gamma \cup \{€\}, €, \delta')$, pri čemu je δ':

- $\delta'(q_0', \varepsilon, \varepsilon) = \{(q_0, \$)\}$,
- $\delta'(q, u, s) = \delta(q, u, s)$, $q \in Q$, $u \in \Sigma \cup \{\varepsilon\}$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q, \varepsilon, €) = \{(q_{\mathrm{f}}', \varepsilon)\}$, $q \in Q$.

$$q_0' \begin{bmatrix} w \\ € \end{bmatrix} \vdash q_0 \begin{bmatrix} w \\ \$€ \end{bmatrix} \vdash_{\mathbb{A}}^{*} q \begin{bmatrix} \varepsilon \\ € \end{bmatrix} \vdash q_{\mathrm{f}}' \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Jezik automata

Dokaz.

Inkluzije $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{e}}(\Sigma)$ i $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{f}}(\Sigma)$ su očigledne.

Dokažimo $\mathcal{L}_{\mathrm{e}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{fe}}(\Sigma)$.

Neka $L \in \mathcal{L}_{\mathrm{e}}(\Sigma)$. Tada postoji potisni automat $\mathbb{A} = (Q, q_0, \emptyset, \Sigma, \Gamma, \$, \delta)$ takav da je $L = L_{\mathrm{e}}(\mathbb{A})$. Nije teško konstruisati potisni automat $\mathbb{A}'$ koji:

- simulira $\mathbb{A}$ i
- kada $\mathbb{A}$ isprazni svoj stek, $\mathbb{A}'$ ulazi u završno stanje.

$\mathbb{A}' = (Q \cup \{q_0', q_{\mathrm{f}}'\}, q_0', \{q_{\mathrm{f}}'\}, \Sigma, \Gamma \cup \{\text{€}\}, \text{€}, \delta')$, pri čemu je δ':

- $\delta'(q_0', \varepsilon, \varepsilon) = \{(q_0, \$)\}$,
- $\delta'(q, u, s) = \delta(q, u, s)$, $q \in Q$, $u \in \Sigma \cup \{\varepsilon\}$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q, \varepsilon, \text{€}) = \{(q_{\mathrm{f}}', \varepsilon)\}$, $q \in Q$.

$$q_0' \begin{bmatrix} w \\ \text{€} \end{bmatrix} \vdash q_0 \begin{bmatrix} w \\ \$\text{€} \end{bmatrix} \vdash^*_{\mathbb{A}} q \begin{bmatrix} \varepsilon \\ \text{€} \end{bmatrix} \vdash q_{\mathrm{f}}' \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Jezik automata

Dokaz.

Inkluzije $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{e}}(\Sigma)$ i $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{f}}(\Sigma)$ su očigledne.

Dokažimo $\mathcal{L}_{\mathrm{e}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{fe}}(\Sigma)$.

Neka $L \in \mathcal{L}_{\mathrm{e}}(\Sigma)$. Tada postoji potisni automat $\mathbb{A} = (Q, q_0, \emptyset, \Sigma, \Gamma, \$, \delta)$ takav da je $L = L_{\mathrm{e}}(\mathbb{A})$. Nije teško konstruisati potisni automat $\mathbb{A}'$ koji:

- simulira $\mathbb{A}$ i
- kada $\mathbb{A}$ isprazni svoj stek, $\mathbb{A}'$ ulazi u završno stanje.

$\mathbb{A}' = (Q \cup \{q_0', q_{\mathrm{f}}'\}, q_0', \{q_{\mathrm{f}}'\}, \Sigma, \Gamma \cup \{€\}, €, \delta')$, pri čemu je δ':

- $\delta'(q_0', \varepsilon, \varepsilon) = \{(q_0, \$)\}$,
- $\delta'(q, u, s) = \delta(q, u, s)$, $q \in Q$, $u \in \Sigma \cup \{\varepsilon\}$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q, \varepsilon, €) = \{(q_{\mathrm{f}}', \varepsilon)\}$, $q \in Q$.

$$q_0' \begin{bmatrix} w \\ € \end{bmatrix} \vdash q_0 \begin{bmatrix} w \\ \$€ \end{bmatrix} \vdash_{\mathbb{A}}^{*} q \begin{bmatrix} \varepsilon \\ € \end{bmatrix} \vdash q_{\mathrm{f}}' \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Jezik automata

Dokaz.

Inkluzije $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{e}}(\Sigma)$ i $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{f}}(\Sigma)$ su očigledne.

Dokažimo $\mathcal{L}_{\mathrm{e}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{fe}}(\Sigma)$.

Neka $L \in \mathcal{L}_{\mathrm{e}}(\Sigma)$. Tada postoji potisni automat $\mathbb{A} = (Q, q_0, \emptyset, \Sigma, \Gamma, \$, \delta)$ takav da je $L = L_{\mathrm{e}}(\mathbb{A})$. Nije teško konstruisati potisni automat $\mathbb{A}'$ koji:

- simulira $\mathbb{A}$ i
- kada $\mathbb{A}$ isprazni svoj stek, $\mathbb{A}'$ ulazi u završno stanje.

$\mathbb{A}' = (Q \cup \{q_0', q_{\mathrm{f}}'\}, q_0', \{q_{\mathrm{f}}'\}, \Sigma, \Gamma \cup \{€\}, €, \delta')$, pri čemu je δ':

- $\delta'(q_0', \varepsilon, \varepsilon) = \{(q_0, \$)\}$,
- $\delta'(q, u, s) = \delta(q, u, s)$, $q \in Q$, $u \in \Sigma \cup \{\varepsilon\}$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q, \varepsilon, €) = \{(q_{\mathrm{f}}', \varepsilon)\}$, $q \in Q$.

$$q_0' \begin{bmatrix} w \\ € \end{bmatrix} \vdash q_0 \begin{bmatrix} w \\ \$€ \end{bmatrix} \vdash_{\mathbb{A}}^{*} q \begin{bmatrix} \varepsilon \\ € \end{bmatrix} \vdash q_{\mathrm{f}}' \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Jezik automata

Dokaz.

Inkluzije $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{e}}(\Sigma)$ i $\mathcal{L}_{\mathrm{fe}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{f}}(\Sigma)$ su očigledne.

Dokažimo $\mathcal{L}_{\mathrm{e}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{fe}}(\Sigma)$.

Neka $L \in \mathcal{L}_{\mathrm{e}}(\Sigma)$. Tada postoji potisni automat $\mathbb{A} = (Q, q_0, \emptyset, \Sigma, \Gamma, \$, \delta)$ takav da je $L = L_{\mathrm{e}}(\mathbb{A})$. Nije teško konstruisati potisni automat $\mathbb{A}'$ koji:

- simulira $\mathbb{A}$ i
- kada $\mathbb{A}$ isprazni svoj stek, $\mathbb{A}'$ ulazi u završno stanje.

$\mathbb{A}' = (Q \cup \{q_0', q_{\mathrm{f}}'\}, q_0', \{q_{\mathrm{f}}'\}, \Sigma, \Gamma \cup \{€\}, €, \delta')$, pri čemu je δ':

- $\delta'(q_0', \varepsilon, \varepsilon) = \{(q_0, \$)\}$,
- $\delta'(q, u, s) = \delta(q, u, s)$, $q \in Q$, $u \in \Sigma \cup \{\varepsilon\}$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q, \varepsilon, €) = \{(q_{\mathrm{f}}', \varepsilon)\}$, $q \in Q$.

$$q_0' \begin{bmatrix} w \\ € \end{bmatrix} \vdash q_0 \begin{bmatrix} w \\ \$€ \end{bmatrix} \vdash_{\mathbb{A}}^{*} q \begin{bmatrix} \varepsilon \\ € \end{bmatrix} \vdash q_{\mathrm{f}}' \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Jezik automata

Dokaz.

Dokažimo sada $\mathcal{L}_{\mathrm{f}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{fe}}(\Sigma)$.

Neka $L \in \mathcal{L}_{\mathrm{f}}(\Sigma)$. Tada postoji potisni automat $\mathbb{A} = (Q, q_0, F, \Sigma, \Gamma, \$, \delta)$ takav da je $L = L_{\mathrm{f}}(\mathbb{A})$. Nije teško konstruisati potisni automat $\mathbb{A}'$ koji:

- simulira $\mathbb{A}$ i
- kada $\mathbb{A}$ udje u završno stanje, $\mathbb{A}'$ prazni svoj stek.

$\mathbb{A}' = (Q \cup \{q_0', q_{\mathrm{e}}'\}, q_0', \{q_{\mathrm{e}}'\}, \Sigma, \Gamma \cup \{€\}, €, \delta')$, pri čemu je δ' odredjeno uslovima:

- $\delta'(q_0', \varepsilon, \varepsilon) = \{(q_0, \$)\}$,
- $\delta'(q, u, s) = \delta(q, u, s)$, $q \in Q$, $u \in \Sigma \cup \{\varepsilon\}$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q, \varepsilon, s) = \{(q_{\mathrm{e}}', \varepsilon)\}$, $q \in F$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q_{\mathrm{e}}', \varepsilon, s) = \{(q_{\mathrm{e}}', \varepsilon)\}$, $s \in \Gamma \cup \{€\}$.

□

Jezik automata

Dokaz.

Dokažimo sada $\mathcal{L}_{\mathrm{f}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{fe}}(\Sigma)$.

Neka $L \in \mathcal{L}_{\mathrm{f}}(\Sigma)$. Tada postoji potisni automat $\mathbb{A} = (Q, q_0, F, \Sigma, \Gamma, \$, \delta)$ takav da je $L = L_{\mathrm{f}}(\mathbb{A})$. Nije teško konstruisati potisni automat $\mathbb{A}'$ koji:

- simulira $\mathbb{A}$ i
- kada $\mathbb{A}$ udje u završno stanje, $\mathbb{A}'$ prazni svoj stek.

$\mathbb{A}' = (Q \cup \{q_0', q_e'\}, q_0', \{q_e'\}, \Sigma, \Gamma \cup \{\text{€}\}, \text{€}, \delta')$, pri čemu je δ' odredjeno uslovima:

- $\delta'(q_0', \varepsilon, \varepsilon) = \{(q_0, \$)\}$,
- $\delta'(q, u, s) = \delta(q, u, s)$, $q \in Q$, $u \in \Sigma \cup \{\varepsilon\}$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q, \varepsilon, s) = \{(q_e', \varepsilon)\}$, $q \in F$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q_e', \varepsilon, s) = \{(q_e', \varepsilon)\}$, $s \in \Gamma \cup \{\text{€}\}$.

□

Jezik automata

Dokaz.

Dokažimo sada $\mathcal{L}_{\mathrm{f}}(\Sigma) \subseteq \mathcal{L}_{\mathrm{fe}}(\Sigma)$.

Neka $L \in \mathcal{L}_{\mathrm{f}}(\Sigma)$. Tada postoji potisni automat $\mathbb{A} = (Q, q_0, F, \Sigma, \Gamma, \$, \delta)$ takav da je $L = L_{\mathrm{f}}(\mathbb{A})$. Nije teško konstruisati potisni automat $\mathbb{A}'$ koji:

- simulira $\mathbb{A}$ i
- kada $\mathbb{A}$ udje u završno stanje, $\mathbb{A}'$ prazni svoj stek.

$\mathbb{A}' = (Q \cup \{q_0', q_{\mathrm{e}}'\}, q_0', \{q_{\mathrm{e}}'\}, \Sigma, \Gamma \cup \{€\}, €, \delta')$, pri čemu je δ' odredjeno uslovima:

- $\delta'(q_0', \varepsilon, \varepsilon) = \{(q_0, \$)\}$,
- $\delta'(q, u, s) = \delta(q, u, s)$, $q \in Q$, $u \in \Sigma \cup \{\varepsilon\}$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q, \varepsilon, s) = \{(q_{\mathrm{e}}', \varepsilon)\}$, $q \in F$, $s \in \Gamma \cup \{\varepsilon\}$,
- $\delta'(q_{\mathrm{e}}', \varepsilon, s) = \{(q_{\mathrm{e}}', \varepsilon)\}$, $s \in \Gamma \cup \{€\}$.

□

Uopštenje potisnog automata

- Bez gubljenja opštosti možemo pretpostaviti da je funkcija tranzicije potisnih automata oblika

$$\overline{\delta} : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \to \mathcal{P}_{\text{fin}}(Q \times \Gamma^*)$$

- Svaki potisni automat sa ovakvom funkcijom tranzicije se može transformisati u običan potisni automat: ako

$$(q', s_1 \ldots s_\ell) \in \overline{\delta}(q, u, s)$$

onda dodajemo nova stanja $q_1, q_2, \ldots, q_{\ell-1}$ i uzimamo da:

$$\begin{aligned}
&(q_1, s_\ell) \in \delta(q, u, s), \ (q_1, s_\ell) \text{ zamenjuje } (q', s_1 \ldots s_\ell)\\
&\delta(q_1, \varepsilon, \varepsilon) = \{(q_2, s_{\ell-1})\},\\
&\delta(q_2, \varepsilon, \varepsilon) = \{(q_3, s_{\ell-2})\},\\
&\vdots\\
&\delta(q_{\ell-1}, \varepsilon, \varepsilon) = \{(q', s_1)\}.
\end{aligned}$$

- Štaviše, može se pretpostaviti i da su funkcije tranzicije oblika: $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \to \mathcal{P}_{\text{fin}}(Q \times \Gamma^*)$

Uopštenje potisnog automata

- Bez gubljenja opštosti možemo pretpostaviti da je funkcija tranzicije potisnih automata oblika

$$\overline{\delta} : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \rightarrow \mathcal{P}_{\text{fin}}(Q \times \Gamma^*)$$

- Svaki potisni automat sa ovakvom funkcijom tranzicije se može transformisati u običan potisni automat: ako

$$(q', s_1 \dots s_\ell) \in \overline{\delta}(q, u, s)$$

onda dodajemo nova stanja q_1, q_2, ..., $q_{\ell-1}$ i uzimamo da:

$$\begin{array}{l} (q_1, s_\ell) \in \delta(q, u, s),\ (q_1, s_\ell) \text{ zamenjuje } (q', s_1 \dots s_\ell) \\ \delta(q_1, \varepsilon, \varepsilon) = \{(q_2, s_{\ell-1})\}, \\ \delta(q_2, \varepsilon, \varepsilon) = \{(q_3, s_{\ell-2})\}, \\ \vdots \\ \delta(q_{\ell-1}, \varepsilon, \varepsilon) = \{(q', s_1)\}. \end{array}$$

- Štaviše, može se pretpostaviti i da su funkcije tranzicije oblika: $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow \mathcal{P}_{\text{fin}}(Q \times \Gamma^*)$

Uopštenje potisnog automata

- Bez gubljenja opštosti možemo pretpostaviti da je funkcija tranzicije potisnih automata oblika

$$\overline{\delta} : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \to \mathcal{P}_{\text{fin}}(Q \times \Gamma^*)$$

- Svaki potisni automat sa ovakvom funkcijom tranzicije se može transformisati u običan potisni automat: ako

$$(q', s_1 \dots s_\ell) \in \overline{\delta}(q, u, s)$$

  onda dodajemo nova stanja q_1, q_2, $\dots$, $q_{\ell-1}$ i uzimamo da:

$$\begin{aligned}
&(q_1, s_\ell) \in \delta(q, u, s),\ (q_1, s_\ell) \text{ zamenjuje } (q', s_1 \dots s_\ell)\\
&\delta(q_1, \varepsilon, \varepsilon) = \{(q_2, s_{\ell-1})\},\\
&\delta(q_2, \varepsilon, \varepsilon) = \{(q_3, s_{\ell-2})\},\\
&\vdots\\
&\delta(q_{\ell-1}, \varepsilon, \varepsilon) = \{(q', s_1)\}.
\end{aligned}$$

- Štaviše, može se pretpostaviti i da su funkcije tranzicije oblika: $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \to \mathcal{P}_{\text{fin}}(Q \times \Gamma^*)$

Kontekstno-slobodni jezici i potisni automati

Теорема

Za svaku kontekstno slobodnu gramatiku $G = (V, \Sigma, S, \mathcal{P})$ *postoji potisni automat* $\mathbb{A}$ *takav da je* $L(G) = L(\mathbb{A})$.

Dokaz.

$\mathbb{A} = (\{q_{\mathrm{start}}, q_{\mathrm{petlja}}, q_{\mathrm{stop}}\}, q_{\mathrm{start}}, \{q_{\mathrm{stop}}\}, \Sigma, V \cup \Sigma \cup \{\$\}, \$, \bar{\delta})$

$\bar{\delta}(q_{\mathrm{start}}, \varepsilon, \$) = \{(q_{\mathrm{petlja}}, S\$)\}$

$\bar{\delta}(q_{\mathrm{petlja}}, \varepsilon, X) = \{(q_{\mathrm{petlja}}, w) \mid X \to w \in \mathcal{P}\}$

$\bar{\delta}(q_{\mathrm{petlja}}, a, a) = \{(q_{\mathrm{petlja}}, \varepsilon)\},\ a \in \Sigma$

$\bar{\delta}(q_{\mathrm{petlja}}, \varepsilon, \$) = \{(q_{\mathrm{stop}}, \varepsilon)\}$

Kontekstno-slobodni jezici i potisni automati

Теорема

Za svaku kontekstno slobodnu gramatiku $G = (V, \Sigma, S, \mathcal{P})$ *postoji potisni automat* $\mathbb{A}$ *takav da je* $L(G) = L(\mathbb{A})$.

Dokaz.

$$\mathbb{A} = (\{q_{\text{start}}, q_{\text{petlja}}, q_{\text{stop}}\}, q_{\text{start}}, \{q_{\text{stop}}\}, \Sigma, V \cup \Sigma \cup \{\$\}, \$, \overline{\delta})$$

$$\overline{\delta}(q_{\text{start}}, \varepsilon, \$) = \{(q_{\text{petlja}}, S\$)\}$$

$$\overline{\delta}(q_{\text{petlja}}, \varepsilon, X) = \{(q_{\text{petlja}}, w) \mid X \to w \in \mathcal{P}\}$$

$$\overline{\delta}(q_{\text{petlja}}, a, a) = \{(q_{\text{petlja}}, \varepsilon)\},\ a \in \Sigma$$

$$\overline{\delta}(q_{\text{petlja}}, \varepsilon, \$) = \{(q_{\text{stop}}, \varepsilon)\}$$

Kontekstno-slobodni jezici i potisni automati

Теорема

Za svaku kontekstno slobodnu gramatiku $G = (V, \Sigma, S, \mathcal{P})$ postoji potisni automat $\mathbb{A}$ takav da je $L(G) = L(\mathbb{A})$.

Dokaz.

$\mathbb{A} = (\{q_{\text{start}}, q_{\text{petlja}}, q_{\text{stop}}\}, q_{\text{start}}, \{q_{\text{stop}}\}, \Sigma, V \cup \Sigma \cup \{\$\}, \$, \overline{\delta})$

$\overline{\delta}(q_{\text{start}}, \varepsilon, \$) = \{(q_{\text{petlja}}, S\$)\}$

$\overline{\delta}(q_{\text{petlja}}, \varepsilon, X) = \{(q_{\text{petlja}}, w) \mid X \to w \in \mathcal{P}\}$

$\overline{\delta}(q_{\text{petlja}}, a, a) = \{(q_{\text{petlja}}, \varepsilon)\}, \ a \in \Sigma$

$\overline{\delta}(q_{\text{petlja}}, \varepsilon, \$) = \{(q_{\text{stop}}, \varepsilon)\}$

Kontekstno-slobodni jezici i potisni automati

Теорема

Za svaku kontekstno slobodnu gramatiku $G = (V, \Sigma, S, \mathcal{P})$ *postoji potisni automat* $\mathbb{A}$ *takav da je* $L(G) = L(\mathbb{A})$.

Dokaz.

$$\mathbb{A} = (\{q_{\text{start}}, q_{\text{petlja}}, q_{\text{stop}}\}, q_{\text{start}}, \{q_{\text{stop}}\}, \Sigma, V \cup \Sigma \cup \{\$\}, \$, \overline{\delta})$$

$$\overline{\delta}(q_{\text{start}}, \varepsilon, \$) = \{(q_{\text{petlja}}, S\$)\}$$

$$\overline{\delta}(q_{\text{petlja}}, \varepsilon, X) = \{(q_{\text{petlja}}, w) \mid X \to w \in \mathcal{P}\}$$

$$\overline{\delta}(q_{\text{petlja}}, a, a) = \{(q_{\text{petlja}}, \varepsilon)\}, \; a \in \Sigma$$

$$\overline{\delta}(q_{\text{petlja}}, \varepsilon, \$) = \{(q_{\text{stop}}, \varepsilon)\}$$

Kontekstno-slobodni jezici i potisni automati

Теорема

Za svaku kontekstno slobodnu gramatiku $G = (V, \Sigma, S, \mathcal{P})$ *postoji potisni automat* $\mathbb{A}$ *takav da je* $L(G) = L(\mathbb{A})$.

Dokaz.

$$\mathbb{A} = (\{q_{\text{start}}, q_{\text{petlja}}, q_{\text{stop}}\}, q_{\text{start}}, \{q_{\text{stop}}\}, \Sigma, V \cup \Sigma \cup \{\$\}, \$, \overline{\delta})$$

$$\overline{\delta}(q_{\text{start}}, \varepsilon, \$) = \{(q_{\text{petlja}}, S\$)\}$$

$$\overline{\delta}(q_{\text{petlja}}, \varepsilon, X) = \{(q_{\text{petlja}}, w) \mid X \to w \in \mathcal{P}\}$$

$$\overline{\delta}(q_{\text{petlja}}, a, a) = \{(q_{\text{petlja}}, \varepsilon)\},\ a \in \Sigma$$

$$\overline{\delta}(q_{\text{petlja}}, \varepsilon, \$) = \{(q_{\text{stop}}, \varepsilon)\}$$

Kontekstno-slobodni jezici i potisni automati

Теорема

Za svaku kontekstno slobodnu gramatiku $G = (V, \Sigma, S, \mathcal{P})$ postoji potisni automat $\mathbb{A}$ takav da je $L(G) = L(\mathbb{A})$.

Dokaz.

$$\mathbb{A} = (\{q_{\mathrm{start}}, q_{\mathrm{petlja}}, q_{\mathrm{stop}}\}, q_{\mathrm{start}}, \{q_{\mathrm{stop}}\}, \Sigma, V \cup \Sigma \cup \{\$\}, \$, \overline{\delta})$$

$$\overline{\delta}(q_{\mathrm{start}}, \varepsilon, \$) = \{(q_{\mathrm{petlja}}, S\$)\}$$

$$\overline{\delta}(q_{\mathrm{petlja}}, \varepsilon, X) = \{(q_{\mathrm{petlja}}, w) \mid X \to w \in \mathcal{P}\}$$

$$\overline{\delta}(q_{\mathrm{petlja}}, a, a) = \{(q_{\mathrm{petlja}}, \varepsilon)\},\ a \in \Sigma$$

$$\overline{\delta}(q_{\mathrm{petlja}}, \varepsilon, \$) = \{(q_{\mathrm{stop}}, \varepsilon)\}$$

Kontekstno-slobodni jezici i potisni automati

Primer.

Konstruišimo potisni automat koji prihvata reči jezika generisanog gramatikom $G = (\{S, A, B\}, \{a, b\}, S, \mathcal{P})$ čija su pravila:

$$\mathcal{P}: \quad S \to a \mid aAB,\ A \to aA \mid a,\ B \to bB \mid b.$$

$S \to aAB \to aaB \to aabB \to aabb$

$\mathbb{A} = (\{q_0, q_\mathrm{p}, q_\mathrm{f}\}, q_0, \{q_\mathrm{f}\}, \{a, b\}, \{S, A, B, a, b, \$\}, \$, \delta)$, gde je δ definisano na sledeći način:

$\delta(q_0, \varepsilon, \$) = \{(q_p, S\$)\}$
$\delta(q_p, \varepsilon, S) = \{(q_\mathrm{p}, a), (q_\mathrm{p}, aAB)\}$
$\delta(q_\mathrm{p}, \varepsilon, A) = \{(q_\mathrm{p}, a), (q_\mathrm{p}, aA)\}$
$\delta(q_\mathrm{p}, \varepsilon, B) = \{(q_\mathrm{p}, b), (q_\mathrm{p}, bB)\}$
$\delta(q_\mathrm{p}, a, a) = \{(q_\mathrm{p}, \varepsilon)\}$
$\delta(q_\mathrm{p}, b, b) = \{(q_\mathrm{p}, \varepsilon)\}$
$\delta(q_\mathrm{p}, \varepsilon, \$) = \{(q_\mathrm{f}, \varepsilon)\}$.

$$q_0 \begin{bmatrix} aabb \\ \$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} aabb \\ S\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} aabb \\ aAB\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} abb \\ AB\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} abb \\ aB\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} bb \\ B\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} bb \\ bB\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} b \\ B\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} b \\ b\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_\mathrm{s} \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Kontekstno-slobodni jezici i potisni automati

Primer.

Konstruišimo potisni automat koji prihvata reči jezika generisanog gramatikom $G = (\{S, A, B\}, \{a, b\}, S, \mathcal{P})$ čija su pravila:

$$\mathcal{P}: \quad S \to a \mid aAB,\ A \to aA \mid a,\ B \to bB \mid b.$$

$S \to aAB \to aaB \to aabB \to aabb$

$\mathbb{A} = (\{q_0, q_p, q_f\}, q_0, \{q_f\}, \{a, b\}, \{S, A, B, a, b, \$\}, \$, \delta)$, gde je δ definisano na sledeći način:

$\delta(q_0, \varepsilon, \$) = \{(q_p, S\$)\}$
$\delta(q_p, \varepsilon, S) = \{(q_p, a), (q_p, aAB)\}$
$\delta(q_p, \varepsilon, A) = \{(q_p, a), (q_p, aA)\}$
$\delta(q_p, \varepsilon, B) = \{(q_p, b), (q_p, bB)\}$
$\delta(q_p, a, a) = \{(q_p, \varepsilon)\}$
$\delta(q_p, b, b) = \{(q_p, \varepsilon)\}$
$\delta(q_p, \varepsilon, \$) = \{(q_f, \varepsilon)\}$.

$$q_0 \begin{bmatrix} aabb \\ \$ \end{bmatrix} \vdash q_p \begin{bmatrix} aabb \\ S\$ \end{bmatrix} \vdash q_p \begin{bmatrix} aabb \\ aAB\$ \end{bmatrix}$$
$$\vdash q_p \begin{bmatrix} abb \\ AB\$ \end{bmatrix} \vdash q_p \begin{bmatrix} abb \\ aB\$ \end{bmatrix}$$
$$\vdash q_p \begin{bmatrix} bb \\ B\$ \end{bmatrix} \vdash q_p \begin{bmatrix} bb \\ bB\$ \end{bmatrix} \vdash q_p \begin{bmatrix} b \\ B\$ \end{bmatrix}$$
$$\vdash q_p \begin{bmatrix} b \\ b\$ \end{bmatrix} \vdash q_p \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_s \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Kontekstno-slobodni jezici i potisni automati

Primer.

Konstruišimo potisni automat koji prihvata reči jezika generisanog gramatikom $G = (\{S, A, B\}, \{a, b\}, S, \mathcal{P})$ čija su pravila:

$$\mathcal{P}: \quad S \to a \mid aAB,\ A \to aA \mid a,\ B \to bB \mid b.$$

$S \to aAB \to aaB \to aabB \to aabb$

$\mathbb{A} = (\{q_0, q_\mathrm{p}, q_\mathrm{f}\}, q_0, \{q_\mathrm{f}\}, \{a, b\}, \{S, A, B, a, b, \$\}, \$, \delta)$, gde je δ definisano na sledeći način:

$\delta(q_0, \varepsilon, \$) = \{(q_p, S\$)\}$
$\delta(q_p, \varepsilon, S) = \{(q_\mathrm{p}, a), (q_\mathrm{p}, aAB)\}$
$\delta(q_\mathrm{p}, \varepsilon, A) = \{(q_\mathrm{p}, a), (q_\mathrm{p}, aA)\}$
$\delta(q_\mathrm{p}, \varepsilon, B) = \{(q_\mathrm{p}, b), (q_\mathrm{p}, bB)\}$
$\delta(q_\mathrm{p}, a, a) = \{(q_\mathrm{p}, \varepsilon)\}$
$\delta(q_\mathrm{p}, b, b) = \{(q_\mathrm{p}, \varepsilon)\}$
$\delta(q_\mathrm{p}, \varepsilon, \$) = \{(q_\mathrm{f}, \varepsilon)\}$.

$$q_0 \begin{bmatrix} aabb \\ \$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} aabb \\ S\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} aabb \\ aAB\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} abb \\ AB\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} abb \\ aB\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} bb \\ B\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} bb \\ bB\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} b \\ B\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} b \\ b\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_\mathrm{s} \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Kontekstno-slobodni jezici i potisni automati

Primer.

Konstruišimo potisni automat koji prihvata reči jezika generisanog gramatikom $G = (\{S, A, B\}, \{a, b\}, S, \mathcal{P})$ čija su pravila:

$$\mathcal{P} : \quad S \to a \mid aAB,\ A \to aA \mid a,\ B \to bB \mid b.$$

$S \to aAB \to aaB \to aabB \to aabb$

$\mathbb{A} = (\{q_0, q_\mathrm{p}, q_\mathrm{f}\}, q_0, \{q_\mathrm{f}\}, \{a, b\}, \{S, A, B, a, b, \$\}, \$, \delta)$, gde je δ definisano na sledeći način:

$\delta(q_0, \varepsilon, \$) = \{(q_p, S\$)\}$
$\delta(q_p, \varepsilon, S) = \{(q_\mathrm{p}, a), (q_\mathrm{p}, aAB)\}$
$\delta(q_\mathrm{p}, \varepsilon, A) = \{(q_\mathrm{p}, a), (q_\mathrm{p}, aA)\}$
$\delta(q_\mathrm{p}, \varepsilon, B) = \{(q_\mathrm{p}, b), (q_\mathrm{p}, bB)\}$
$\delta(q_\mathrm{p}, a, a) = \{(q_\mathrm{p}, \varepsilon)\}$
$\delta(q_\mathrm{p}, b, b) = \{(q_\mathrm{p}, \varepsilon)\}$
$\delta(q_\mathrm{p}, \varepsilon, \$) = \{(q_\mathrm{f}, \varepsilon)\}$.

$$q_0 \begin{bmatrix} aabb \\ \$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} aabb \\ S\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} aabb \\ aAB\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} abb \\ AB\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} abb \\ aB\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} bb \\ B\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} bb \\ bB\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} b \\ B\$ \end{bmatrix}$$
$$\vdash q_\mathrm{p} \begin{bmatrix} b \\ b\$ \end{bmatrix} \vdash q_\mathrm{p} \begin{bmatrix} \varepsilon \\ \$ \end{bmatrix} \vdash q_\mathrm{s} \begin{bmatrix} \varepsilon \\ \varepsilon \end{bmatrix}$$

Kontekstno-slobodni jezici i potisni automati

Теорема

Svaki jezik koji prihvata potisni automat je kontekstno slobodan.