The Aanderaa-Karp-Rosenberg conjecture: a new perspective

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By property of a graph we mean a boolean function on the set of all graphs; it is called invariant if relabelling of vertices of a graph does not change the value of the property on it. In order to check a certain property of a graph, one needs to ask a number of questions about edges of a graph.

If a (simple) graph has n vertices, then $m = n(n-1)/2$ is the maximal possible number of its edges. The Aanderaa-Rosenberg conjecture (now proved) states that there exists a positive constant C such that at least Cm questions are needed to check any (non-trivial) monotonic invariant property. A stronger Evasiveness Conjecture (still open) asserts that one can always assume $C = 1$ above. The topological approach developed to attack the last conjecture forms an important part of modern topological combinatorics.

In this talk, we'll focus on a version of the Aanderaa-Karp-Rosenberg (evasiveness) conjecture, in which one considers all non-trivial monotonic properties. We'll introduce all the necessary notions and results along the way to a version of the original conjecture for non-invariant monotonic properties proved by Ayzenberg.