Contact CR-submanifolds in spheres

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The notion of CR-submanifold in Kaehler manifolds was introduced by A. Bejancu in 70's, with the aim of unifying two existing notions, namely
complex and totally real submanifolds in Kaehler manifolds. Since then, the topic was rapidly developed, mainly in two directions:
- Study CR-submanifolds in other almost Hermitian manifolds.
- Find the odd analogue of CR-submanifolds. Thus, the notion of semi-invariant submanifold in Sasakian manifolds was introduced. Later on, the name was changed to contact CR-submanifolds.

A huge interest in the last 20 years was focused on the study of CR-submanifolds of the nearly Kaehler six dimensional unit sphere. Interesting and important properties of such submanifolds were discovered, for example, by M. Antic, M. Djoric, F. Dillen, L. Verstraelen, L. Vrancken.
As the odd dimensional counterpart, contact CR-submanifolds in odd dimensional spheres were, recently, intensively studied.

In this talk we focus on those proper contact CR-submanifolds, which are as closed as possible to totally geodesic ones in the seven dimensional spheres endowed with its canonical structure of a Sasakian space form. We give a complete classification for such a submanifold having dimension 4 and describe the techniques of the study. We have also obtained important results concerning dimension 5 and propose further problems in this direction.

This talk is based on some papers in collaboration with M. Djoric and L. Vrancken, mainly on [1].

Keywords: (contact) CR-submanifold, Sasakian manifolds, minimal submanifolds, (mixed) totally geodesic CR-submanifolds

References:
[1] M. Djoric, M.I. Munteanu, L. Vrancken: Four-dimensional contact CR-submanifolds in S7(1), Mathematische Nachrichten, 290 (2017) 16, 2585-2596.