Chapter 3

Parallel Algorithm Design

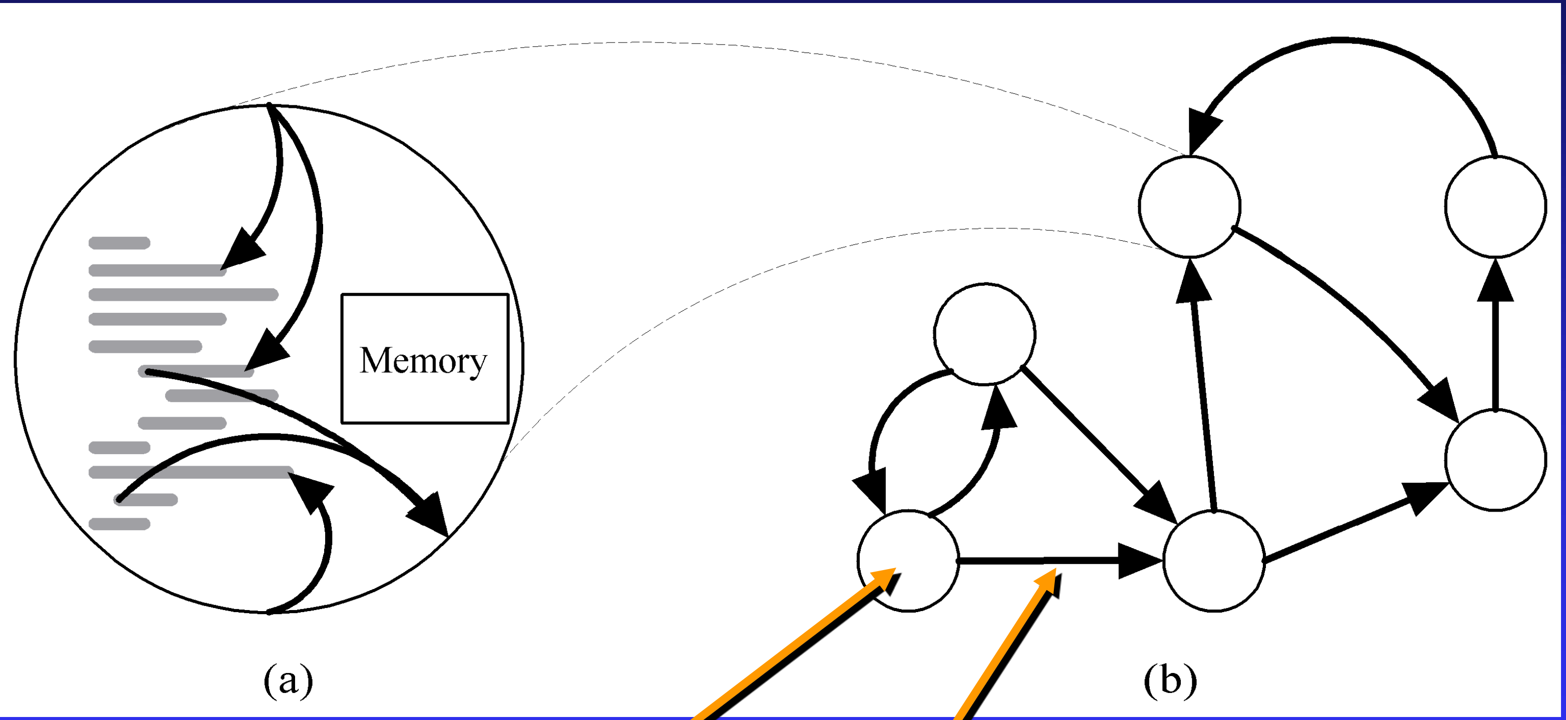
Outline

- Task/channel model
- Algorithm design methodology
- Case studies

Task/Channel Model

- Parallel computation = set of tasks
- Task
  - Program
  - Local memory
  - Collection of I/O ports
- Tasks interact by sending messages through channels

Task/Channel Model



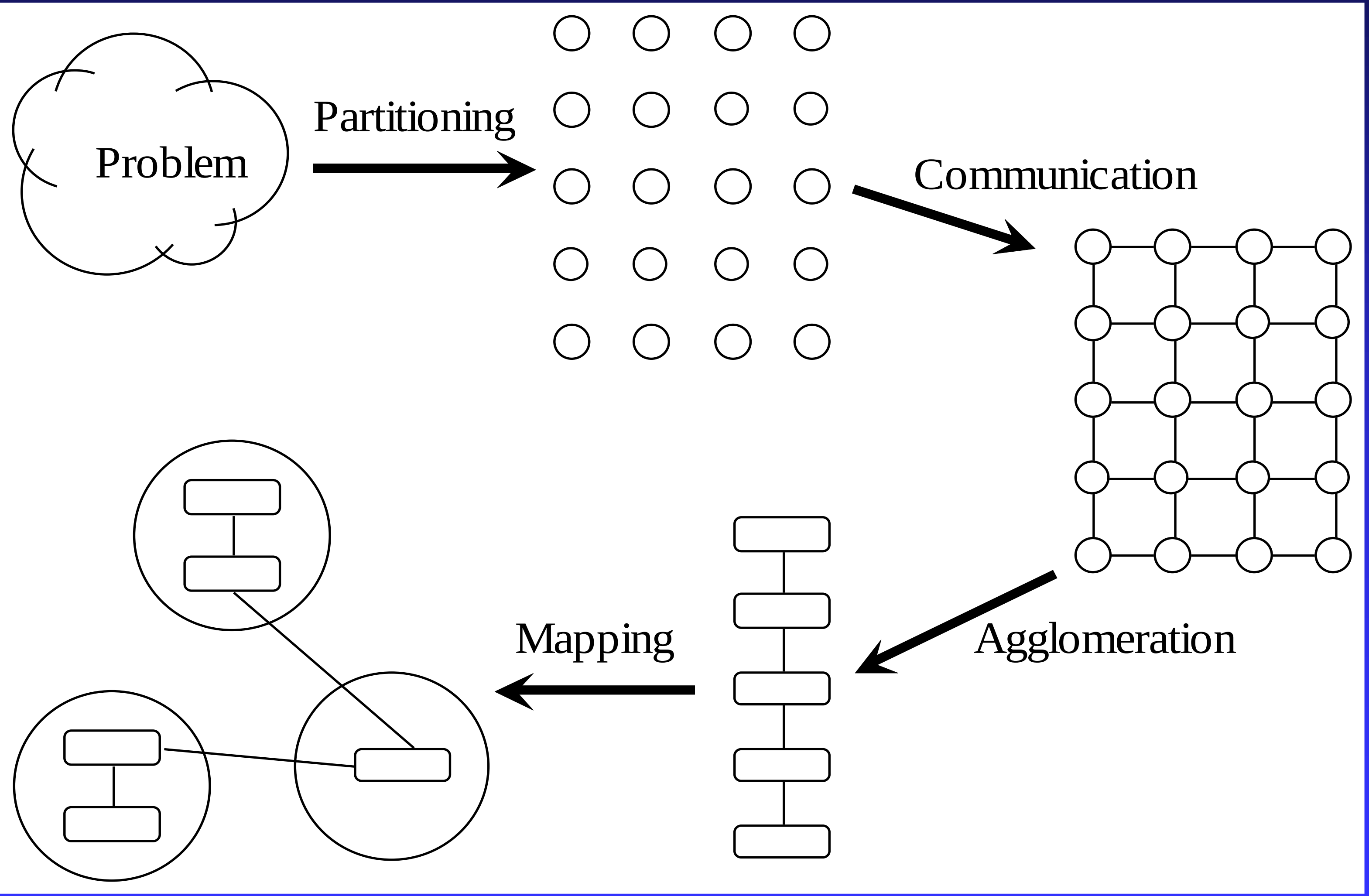
Task

Channel

Foster’s Design Methodology

- Partitioning
- Communication
- Agglomeration
- Mapping

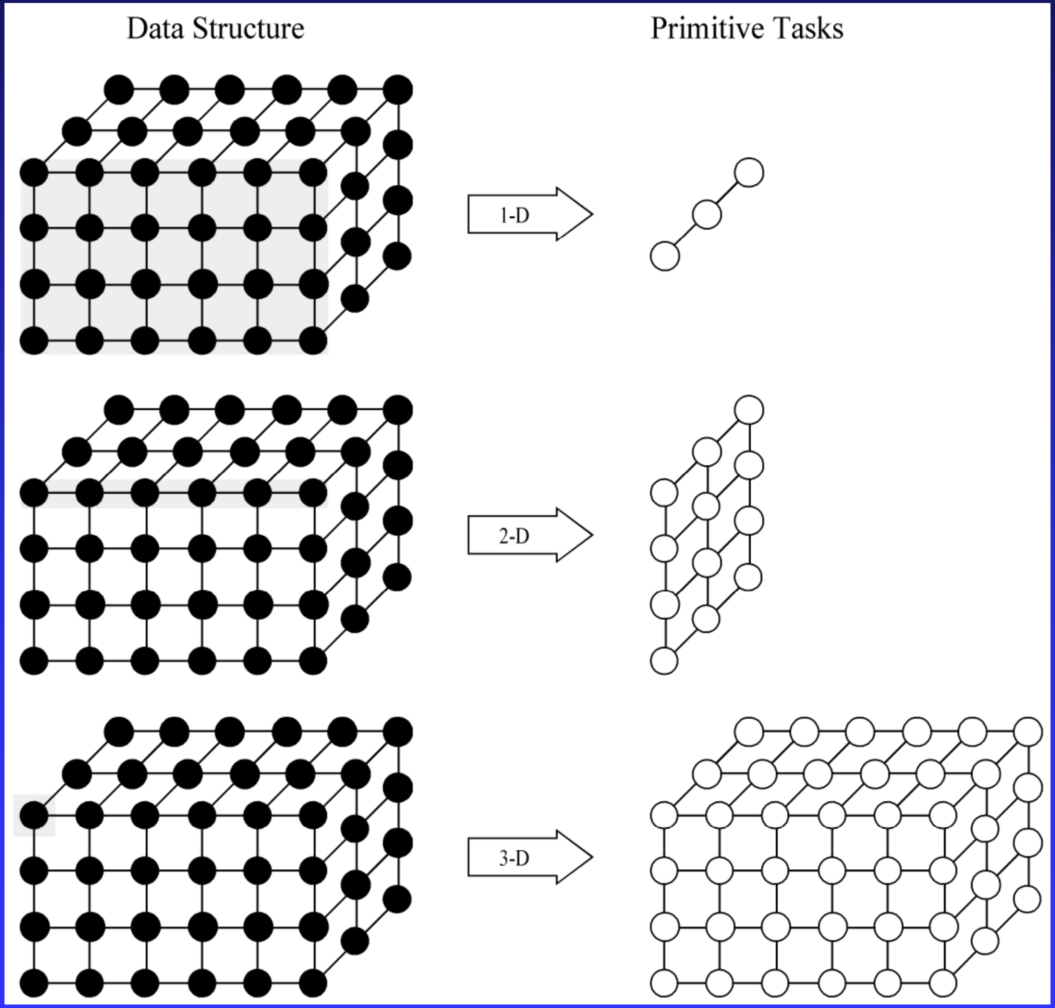
Foster's Methodology



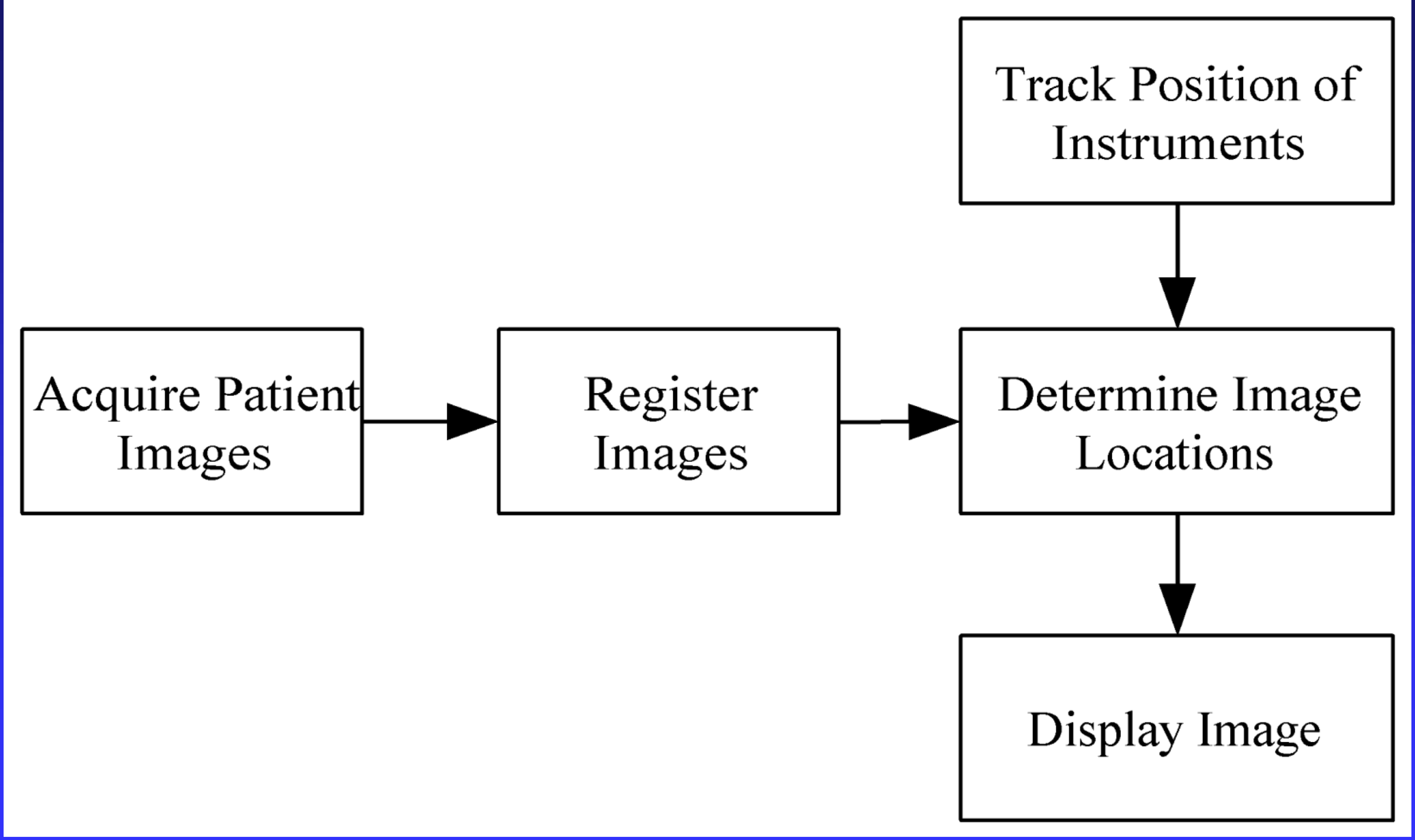
Partitioning

- Dividing computation and data into pieces
- Domain decomposition
  - Divide data into pieces
    - e.g., An array into sub-arrays (reduction); A loop into sub-loops (matrix multiplication), A search space into sub-spaces (chess)
  - Determine how to associate computations with the data
- Functional decomposition
  - Divide computation into pieces
    - e.g., pipelines (floating point multiplication), workflows (pay roll processing)
  - Determine how to associate data with the computations

Example Domain Decompositions



Example Functional Decomposition



Partitioning Checklist

- Large Grained Tasks
  - e.g, at least 10x more primitive tasks than processors in target computer
- Balance Load
  - Primitive tasks roughly the same size
- Scalable
  - Number of tasks an increasing function of problem size

Communication

- Determine values passed among tasks
- Local communication
  - Task needs values from a small number of other tasks
  - Create channels illustrating data flow
- Global communication
  - Significant number of tasks contribute data to perform a computation
  - Don’t create channels for them early in design

Communication Checklist

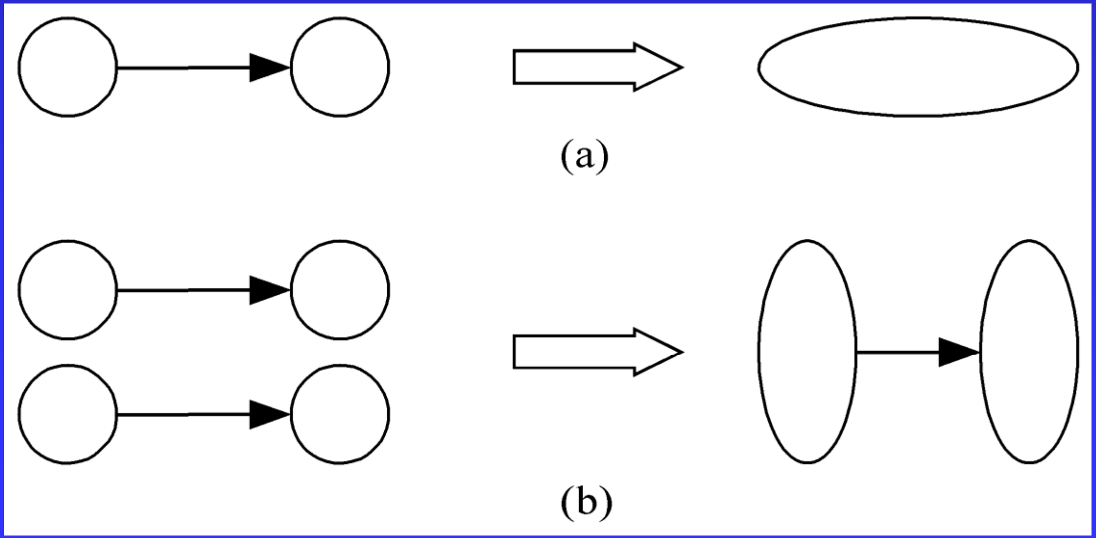
- Balanced
  - Communication operations balanced among tasks
- Small degree:
  - Each task communicates with only small group of neighbors
- Concurrency
  - Tasks can perform communications concurrently
  - Task can perform computations concurrently

Agglomeration

- Grouping tasks into larger tasks
- Goals
  - Improve performance
  - Maintain scalability of program
  - Simplify programming
- In MPI programming, goal often to create one agglomerated task per processor

Agglomeration Can Improve Performance

- Eliminate communication between primitive tasks agglomerated into consolidated task
- Combine groups of sending and receiving tasks



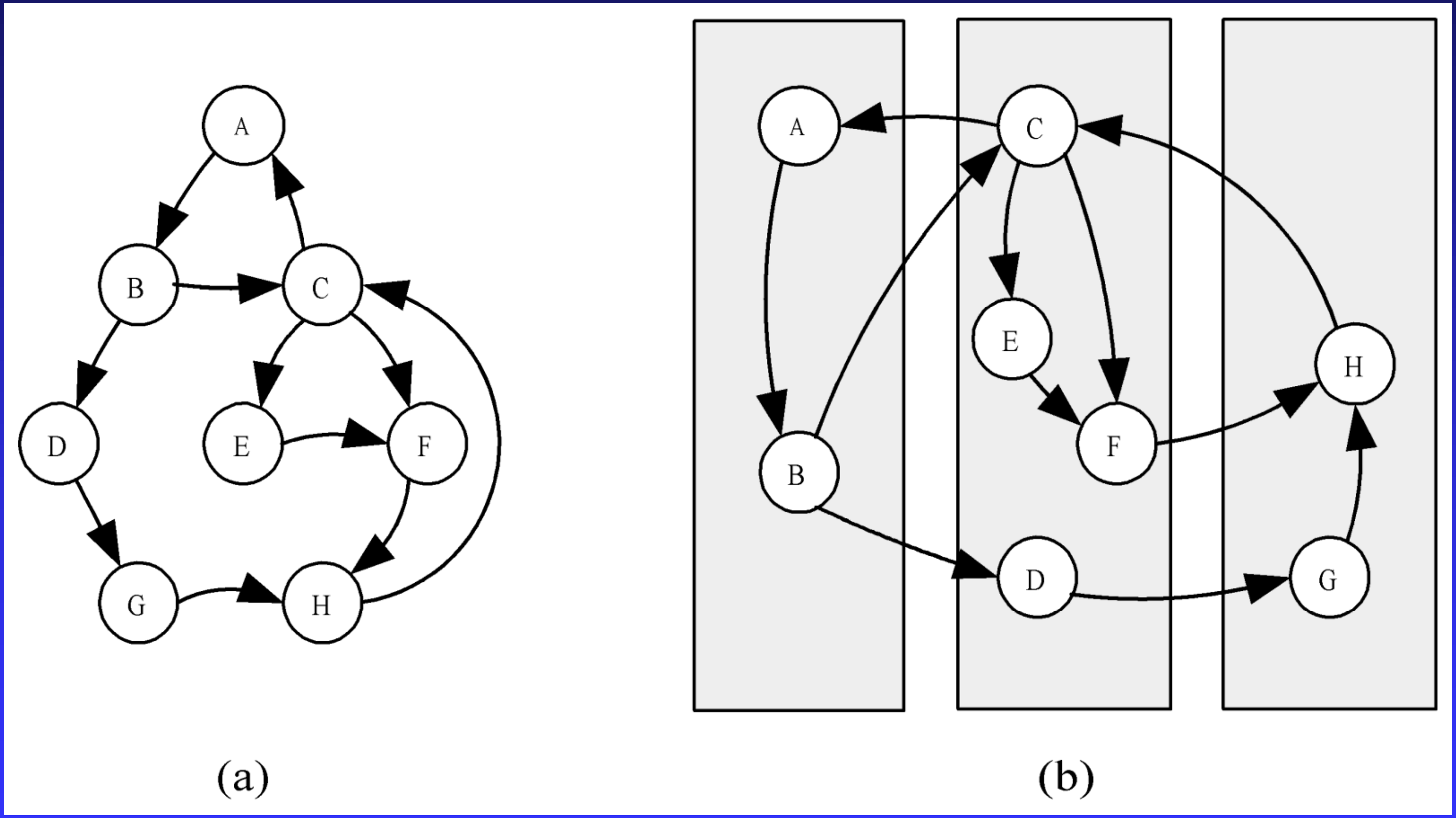
Agglomeration Checklist

- Locality of parallel algorithm has increased
- Tradeoff between agglomeration and code modifications costs is reasonables
- Agglomerated tasks have similar computational and communications costs
- Number of tasks increases with problem size
- Number of tasks suitable for likely target systems

Mapping

- Process of assigning tasks to processors
- Centralized multiprocessor: mapping done by operating system
- Distributed memory system: mapping done by user
- Conflicting goals of mapping
  - Maximize processor utilization
  - Minimize interprocessor communication

Mapping Example



Optimal Mapping

- Finding optimal mapping is NP-hard
- Must rely on heuristics

Mapping Decision Tree

- Static number of tasks
  - Structured communication
    - Constant computation time per task
      - Agglomerate tasks to minimize comm
      - Create one task per processor
    - Variable computation time per task
      - Cyclically map tasks to processors
  - Unstructured communication
      - Use a static load balancing algorithm
- Dynamic number of tasks

Mapping Strategy

- Static number of tasks
- Dynamic number of tasks
  - Use a run-time task-scheduling algorithm
    - e.g., a master slave strategy
  - Use a dynamic load balancing algorithm
    - e.g., share load among neighboring processors; remapping periodically

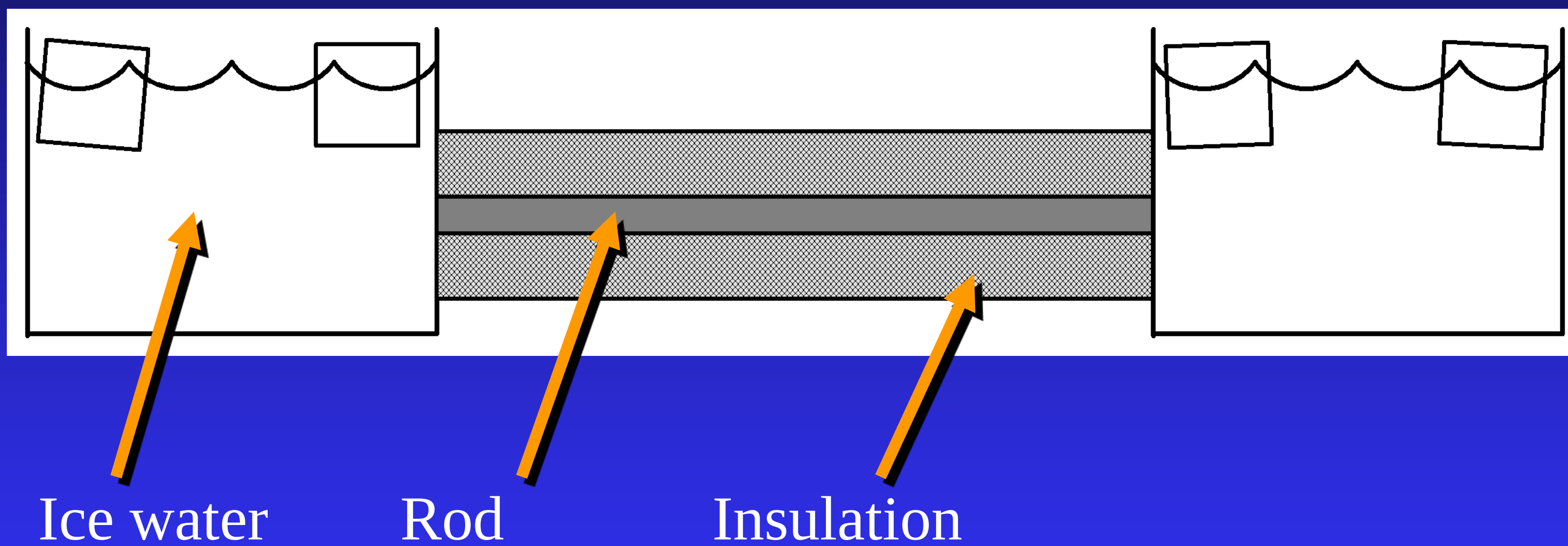
Mapping Checklist

- Considered designs based on one task per processor and multiple tasks per processor
  - If multiple task per processor chosen, ratio of tasks to processors is at least 10:1
- Evaluated static and dynamic task allocation
- If dynamic task allocation chosen, task allocator is not a bottleneck to performance

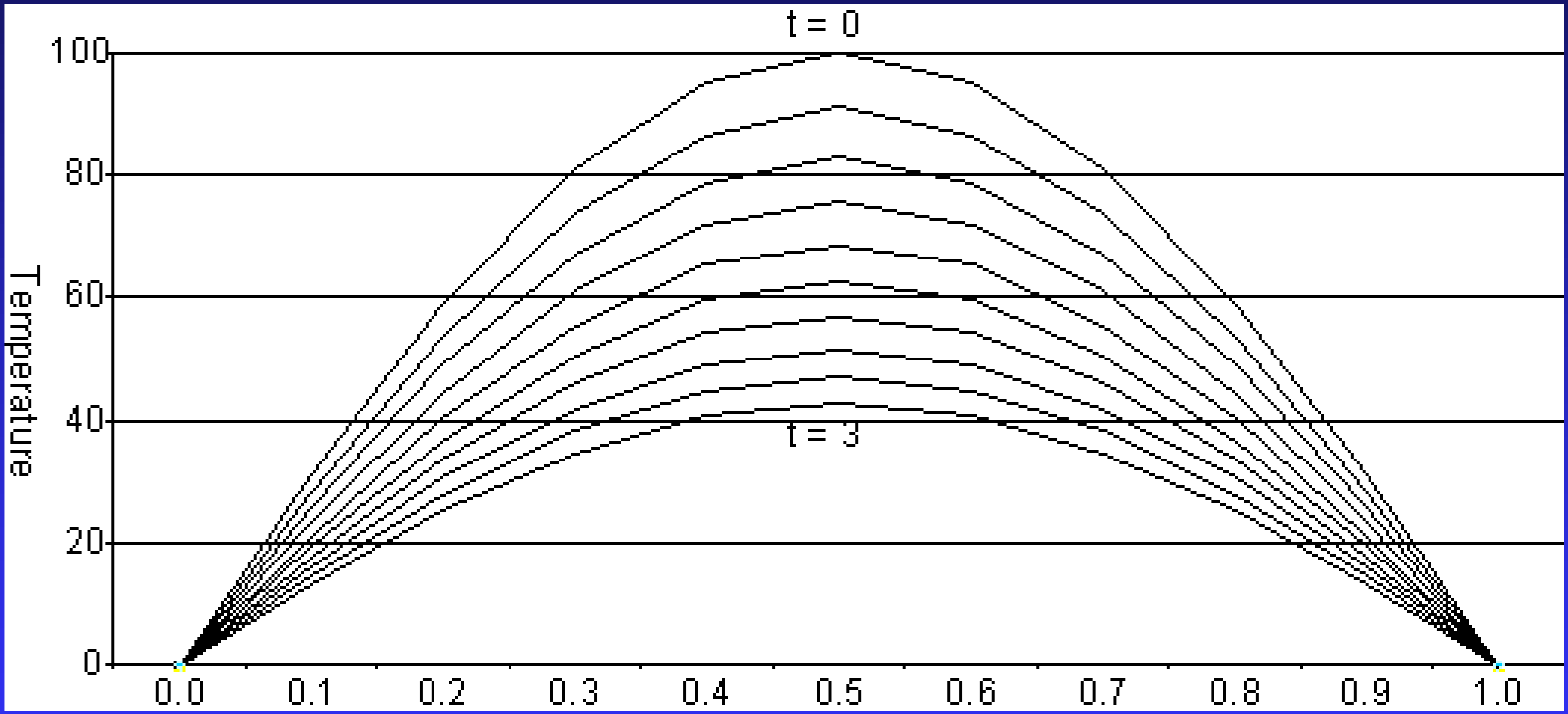
Case Studies

- Boundary value problem
- Finding the maximum
- The n-body problem
- Adding data input

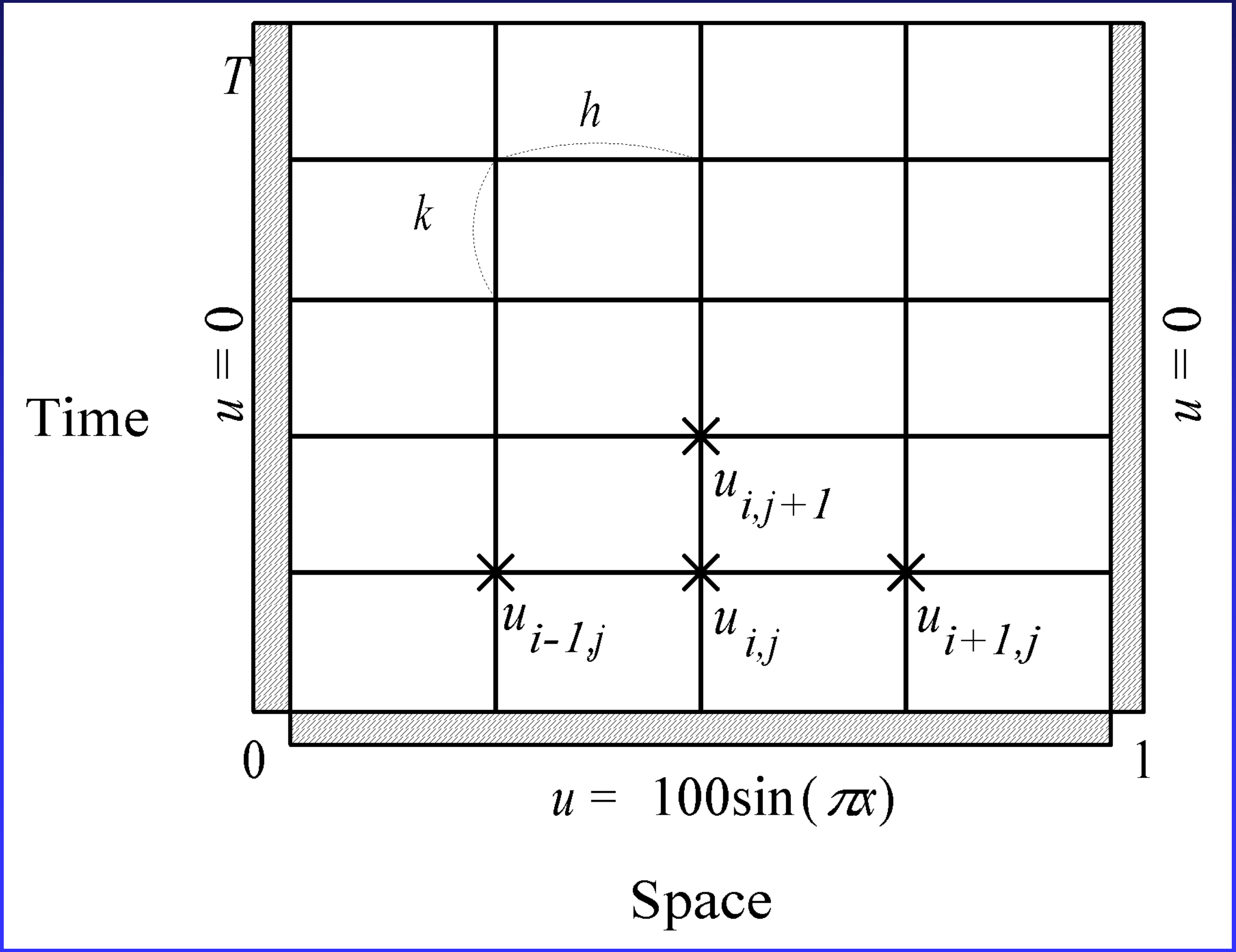
Boundary Value Problem



Rod Cools as Time Progresses



Finite Difference Approximation



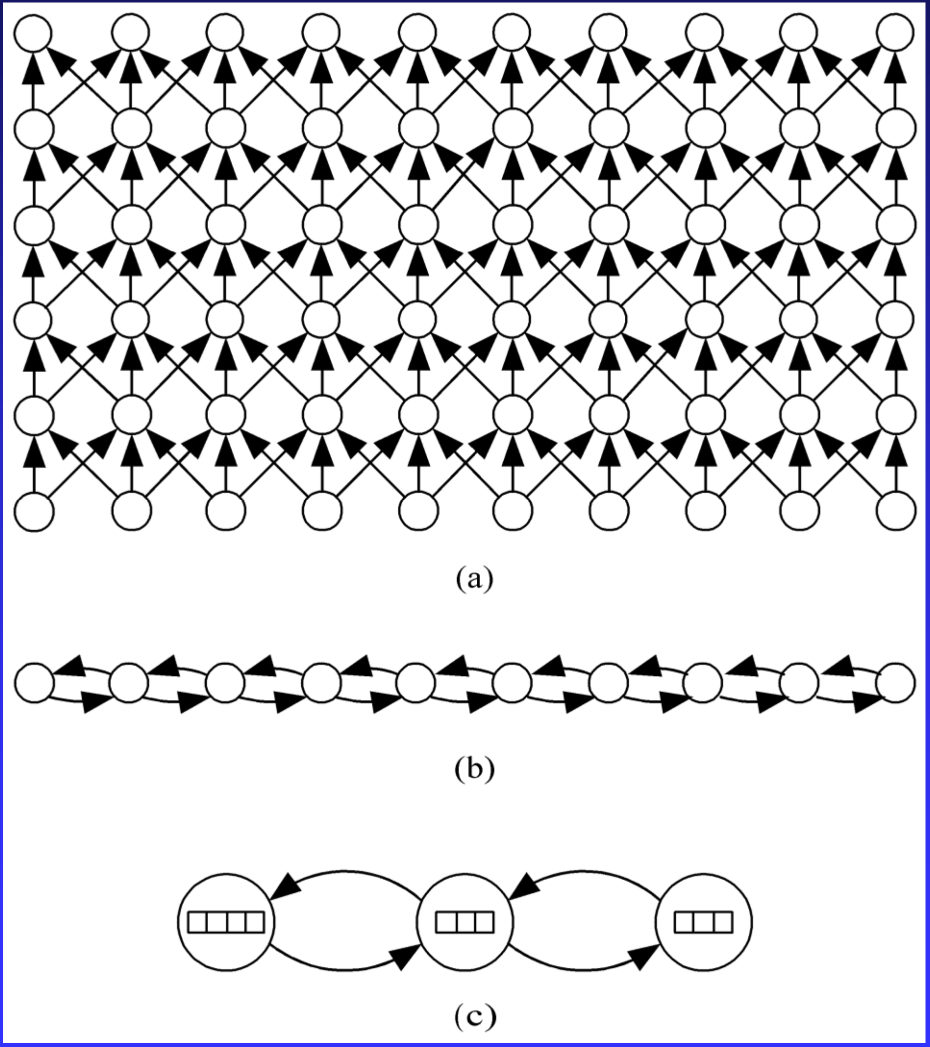
Partitioning

- One data item per grid point
- Associate one primitive task with each grid point
- Two-dimensional domain decomposition

Communication

- Identify communication pattern between primitive tasks
- Each interior primitive task has three incoming and three outgoing channels

Agglomeration and Mapping



Agglomeration

Sequential execution time

- χ – time to update element
- n – number of elements
- m – number of iterations
- Sequential execution time: $mn\chi$

Parallel Execution Time

- p – number of processors
- λ – message latency
- Parallel execution time $m(\chi\lceil n/p \rceil + 2\lambda)$

Finding the Maximum Error

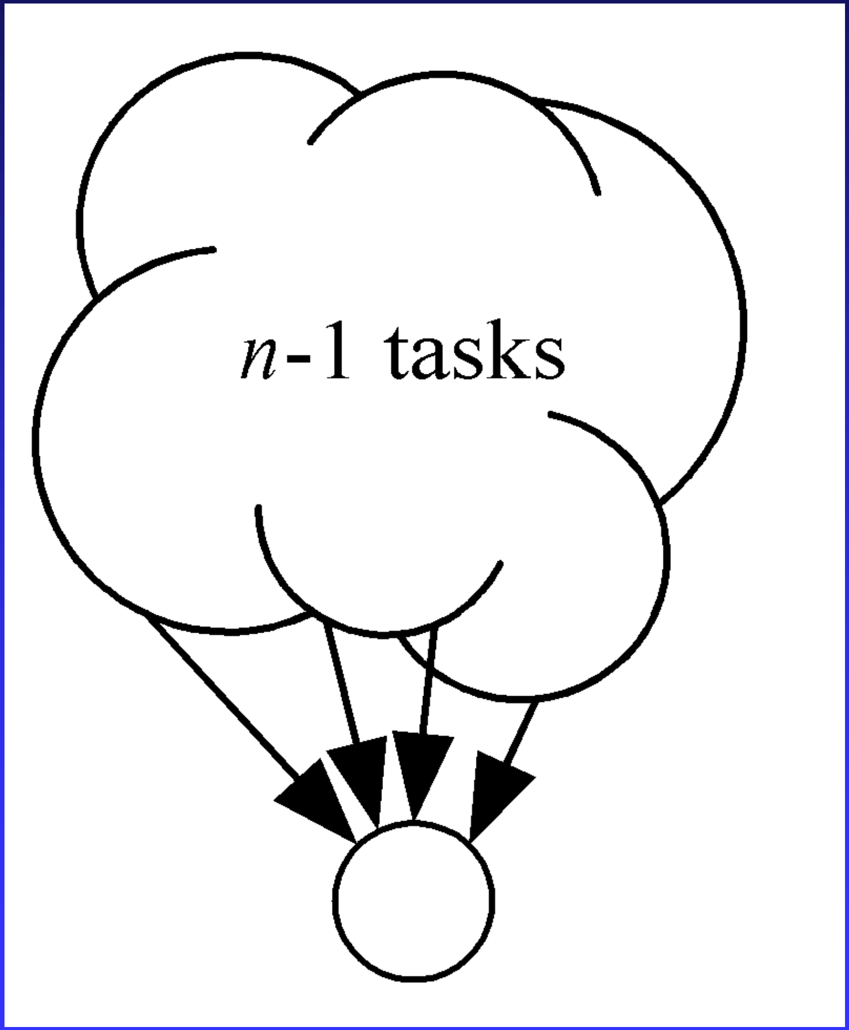
Computed	0.15	0.16	0.16	0.19
Correct	0.15	0.16	0.17	0.18
Error (%)	0.00%	0.00%	6.25%	5.26%

6.25%

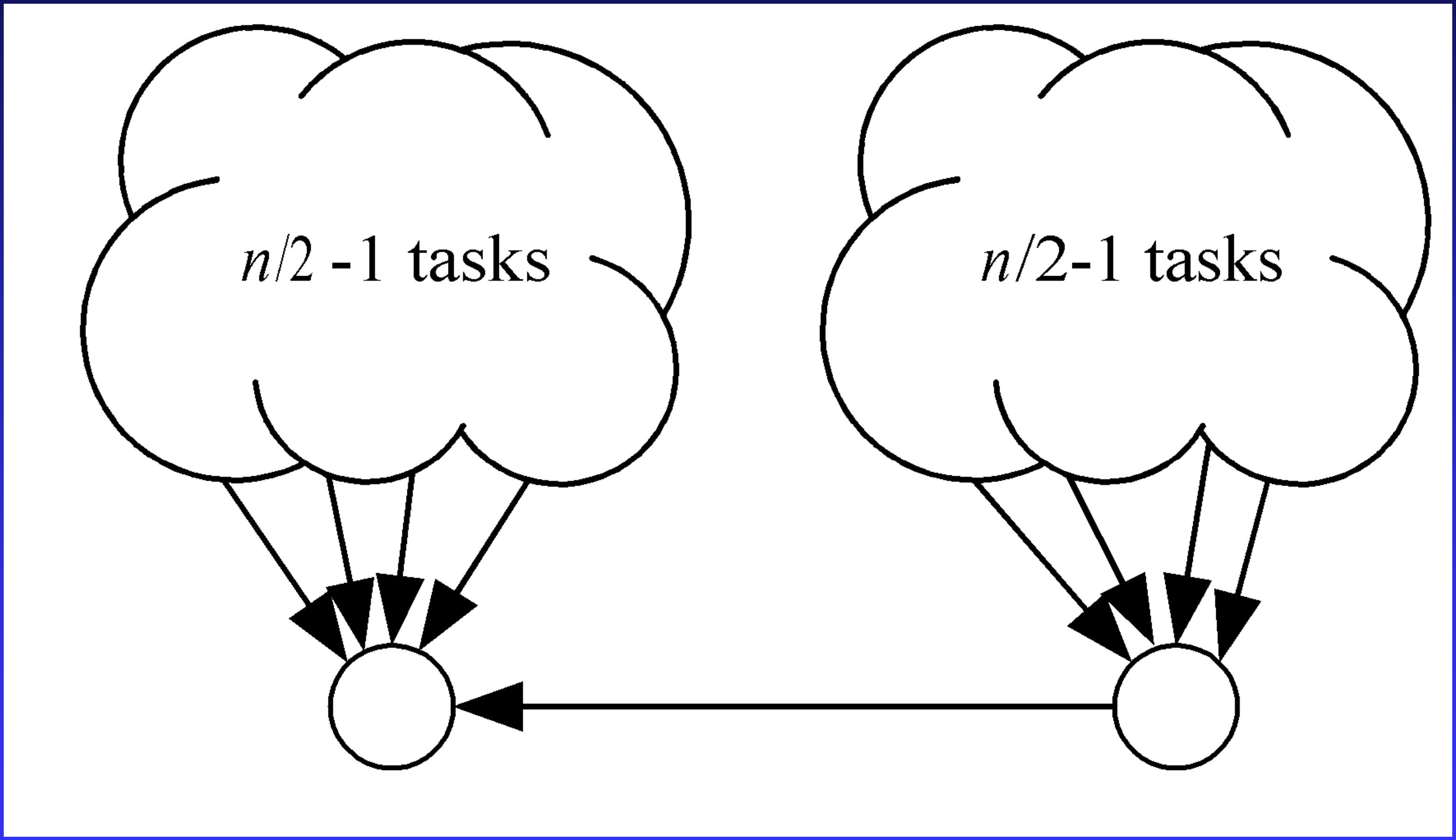
Reduction

- Given associative operator $\oplus$
- $a_0 \oplus a_1 \oplus a_2 \oplus \ldots \oplus a_{n-1}$
- Examples
  - Add
  - Multiply
  - And, Or
  - Maximum, Minimum

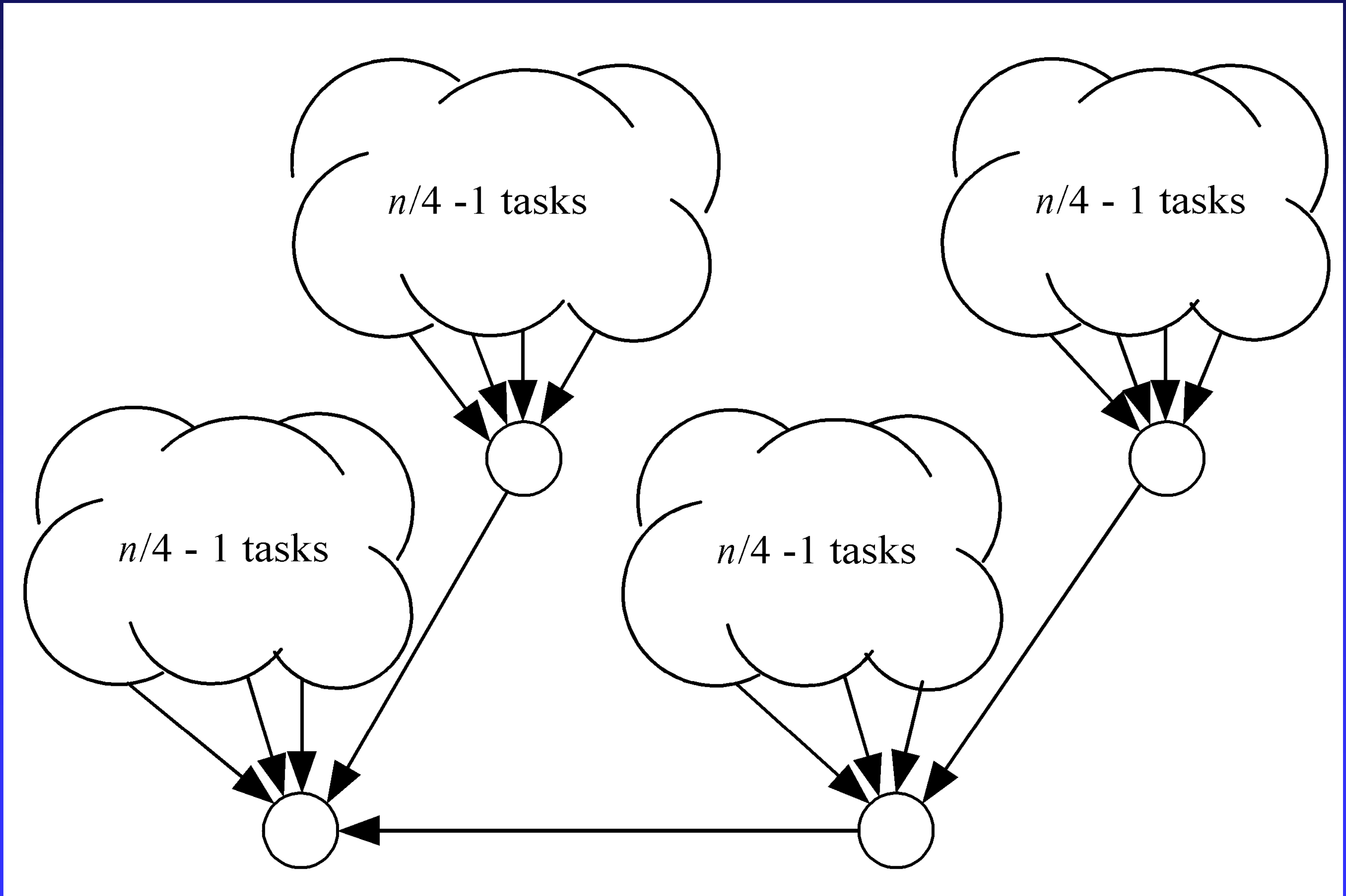
Parallel Reduction Evolution



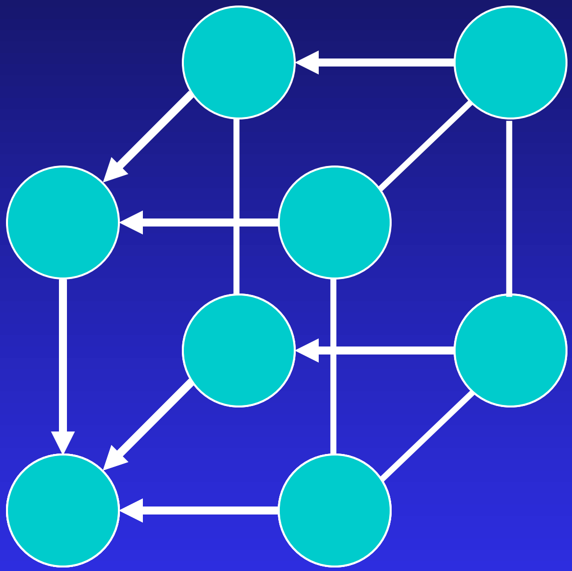
Parallel Reduction Evolution



Parallel Reduction Evolution

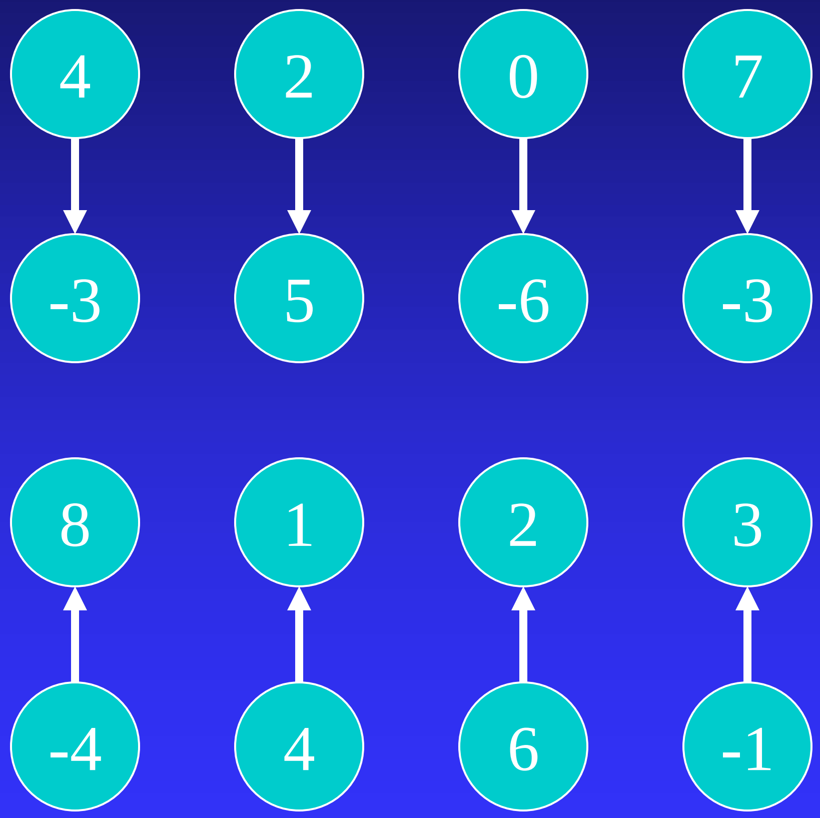


Binomial Trees

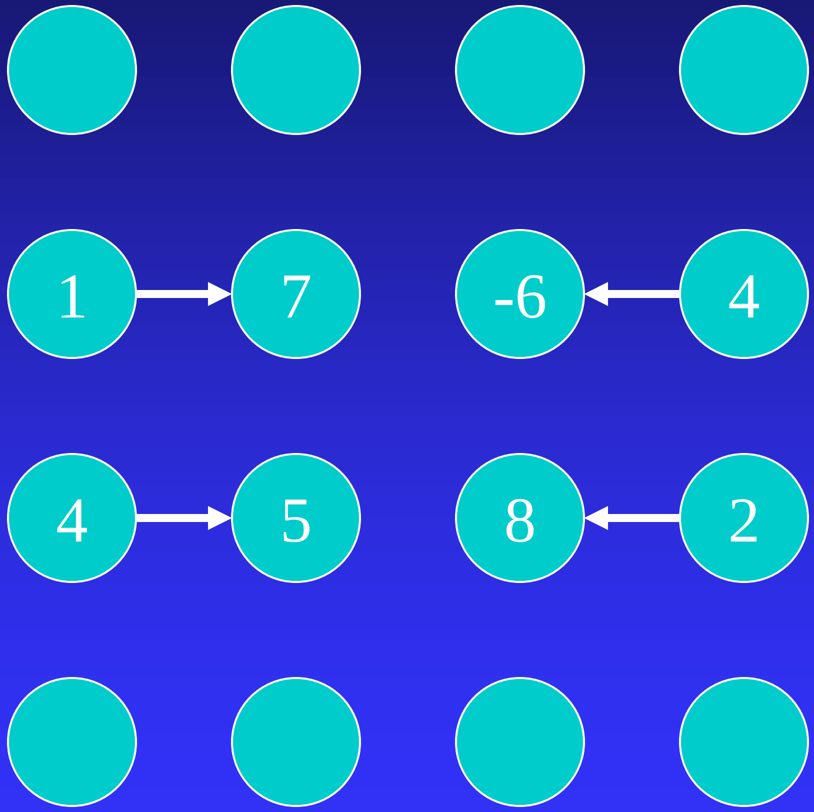


Subgraph of hypercube

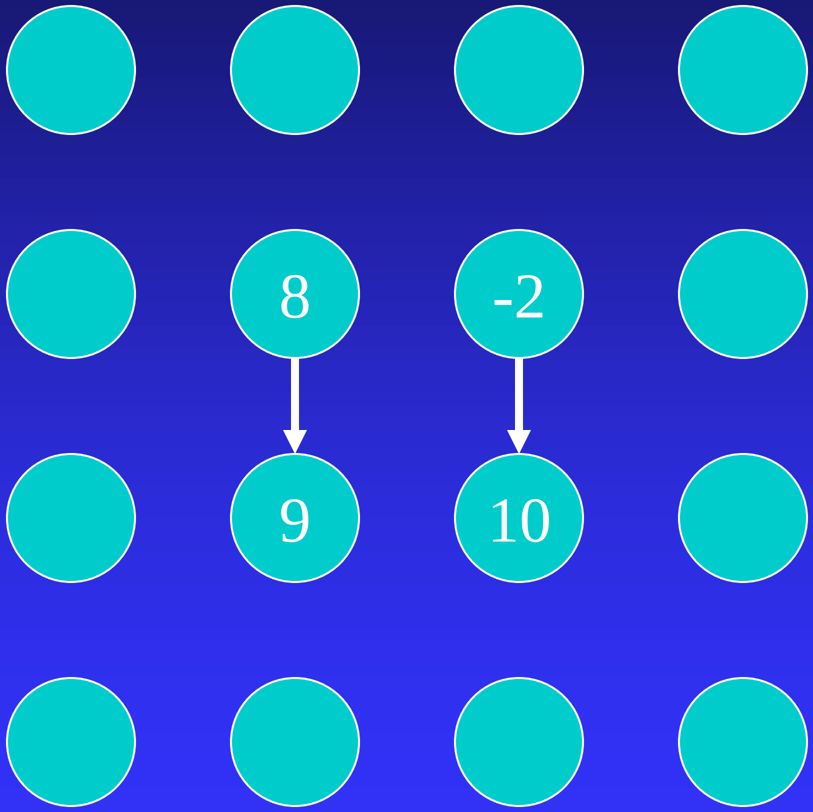
Finding Global Sum



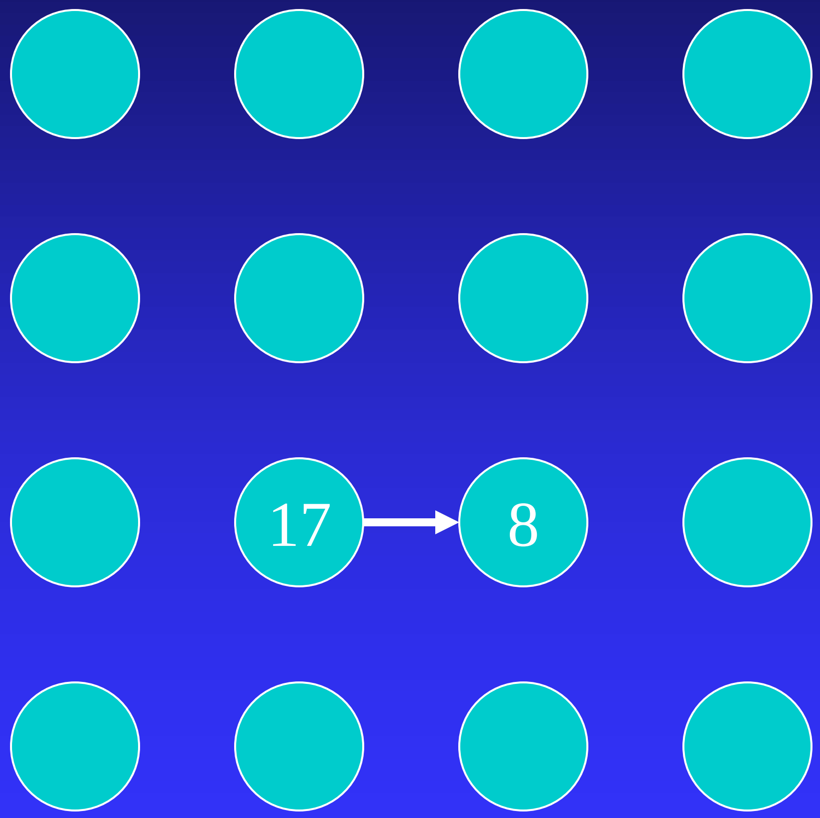
Finding Global Sum



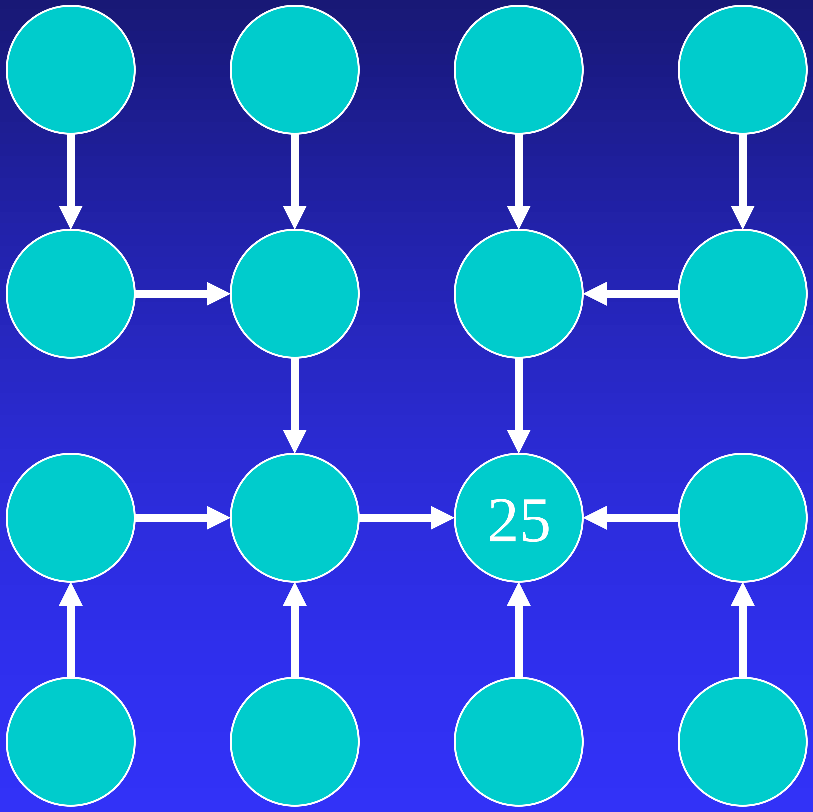
Finding Global Sum



Finding Global Sum

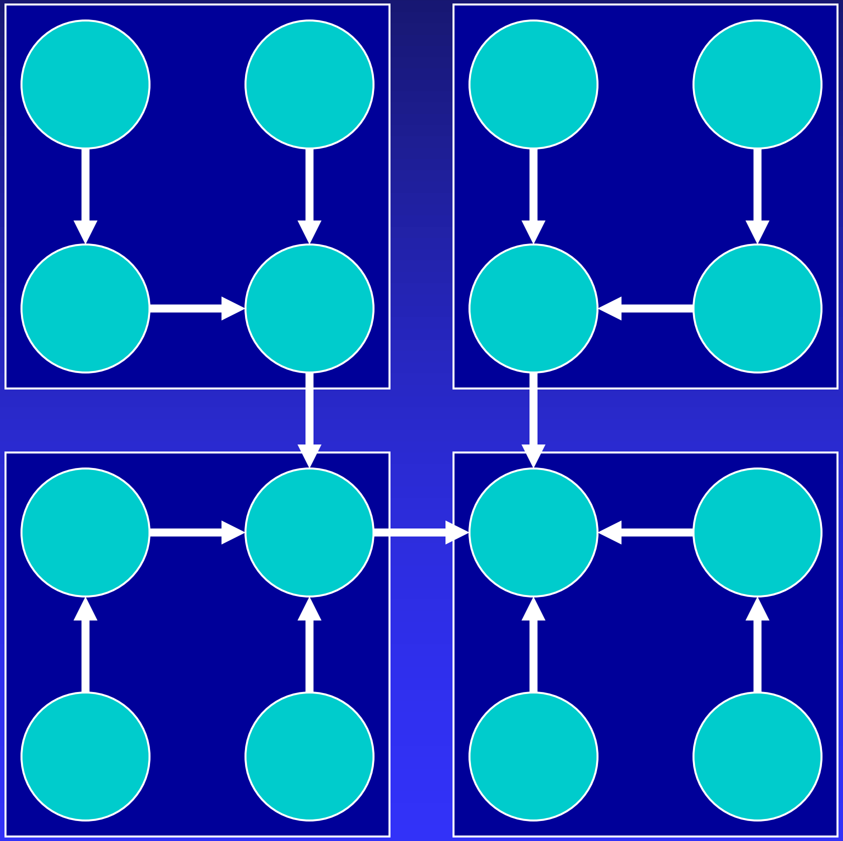


Finding Global Sum

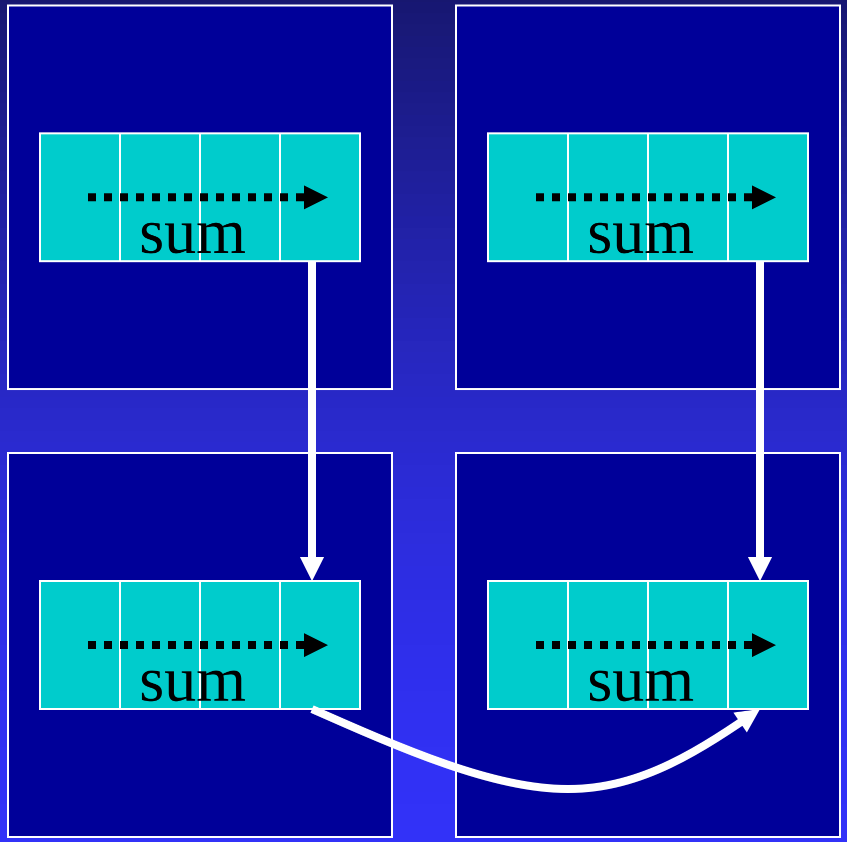


Binomial Tree

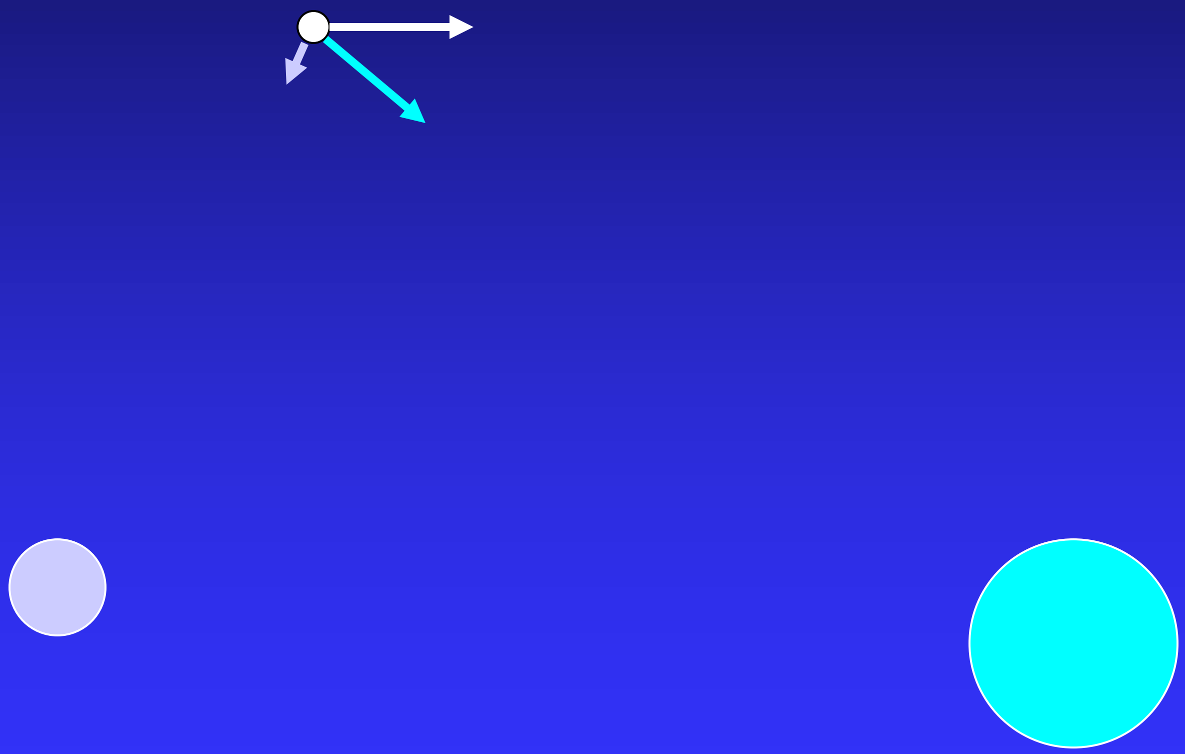
Agglomeration



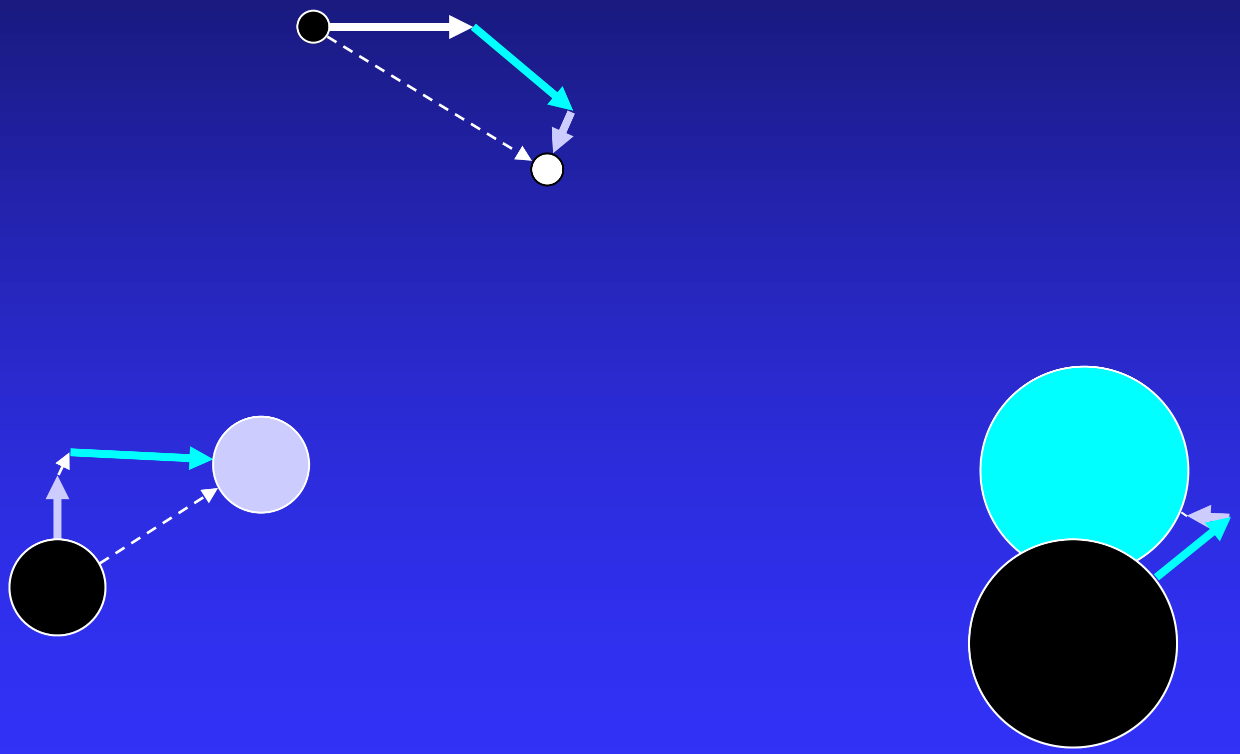
Agglomeration



The n-body Problem



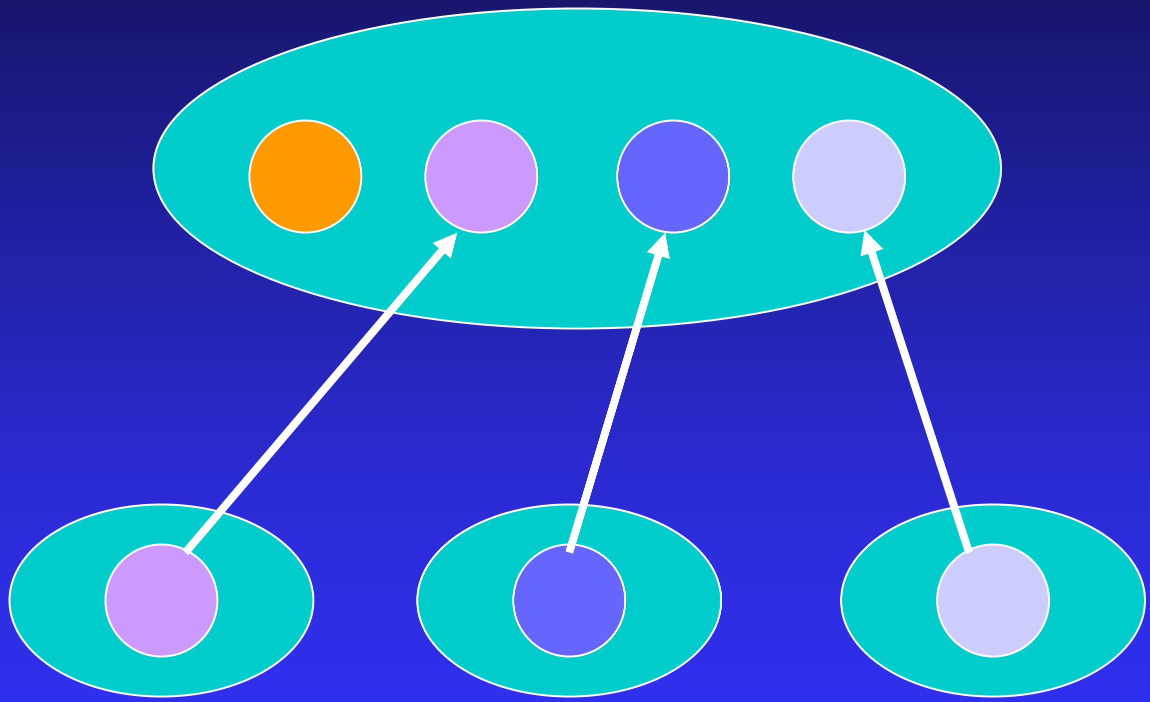
The n-body Problem



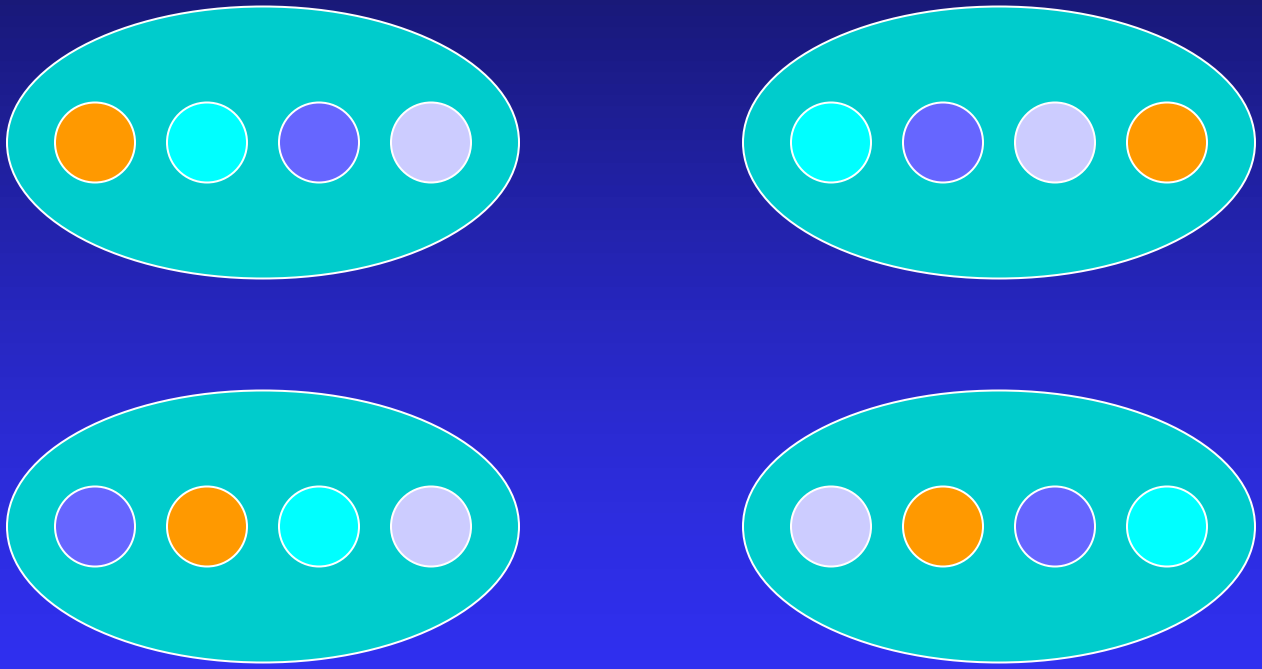
Partitioning

- Domain partitioning
- Assume one task per particle
- Task has particle's position, velocity vector
- Iteration
  - Get positions of all other particles
  - Compute new position, velocity

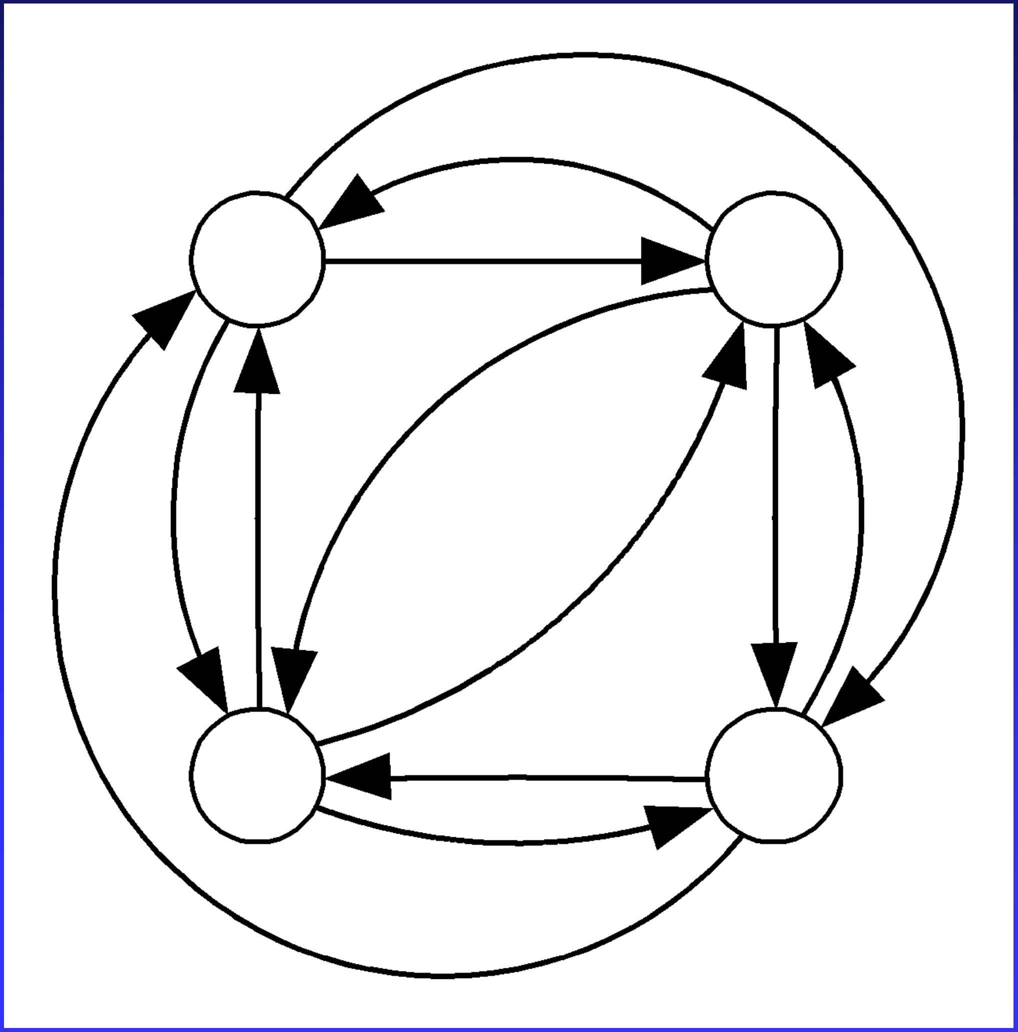
Gather



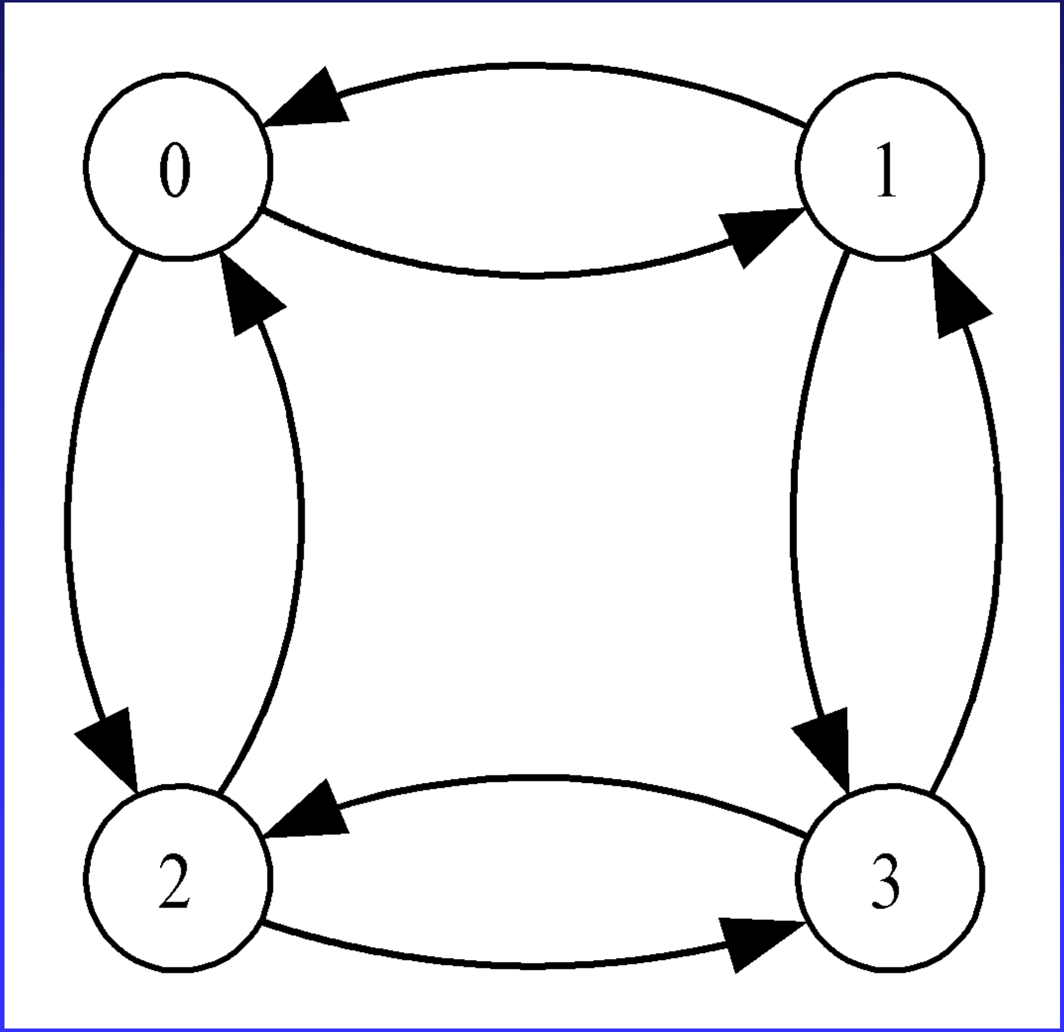
All-gather



Complete Graph for All-gather



Hypercube for All-gather



Communication Time

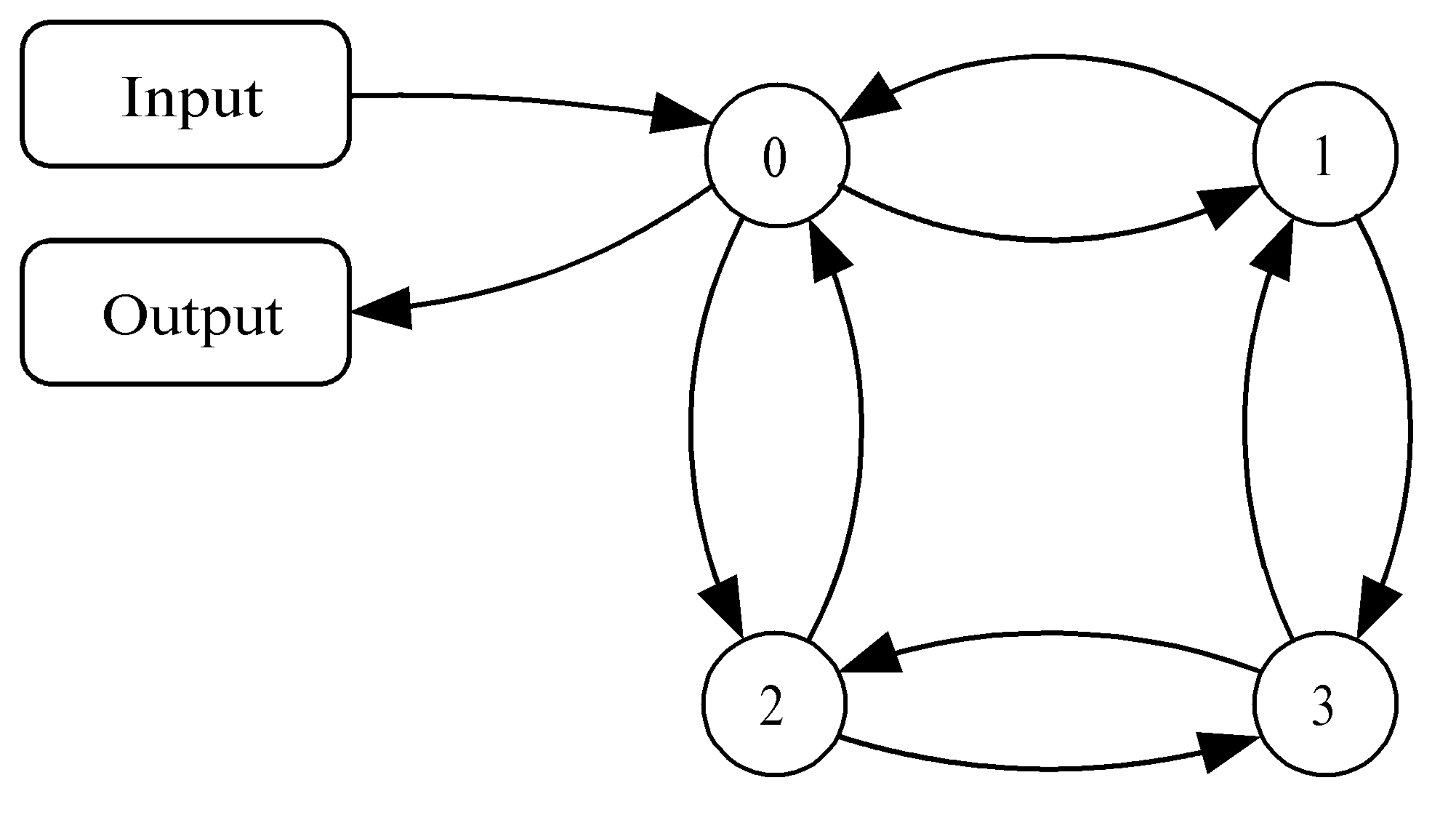
Complete graph

$$(p-1)\left(\lambda+\frac{n/p}{\beta}\right)=(p-1)\lambda+\frac{n(p-1)}{\beta p}$$

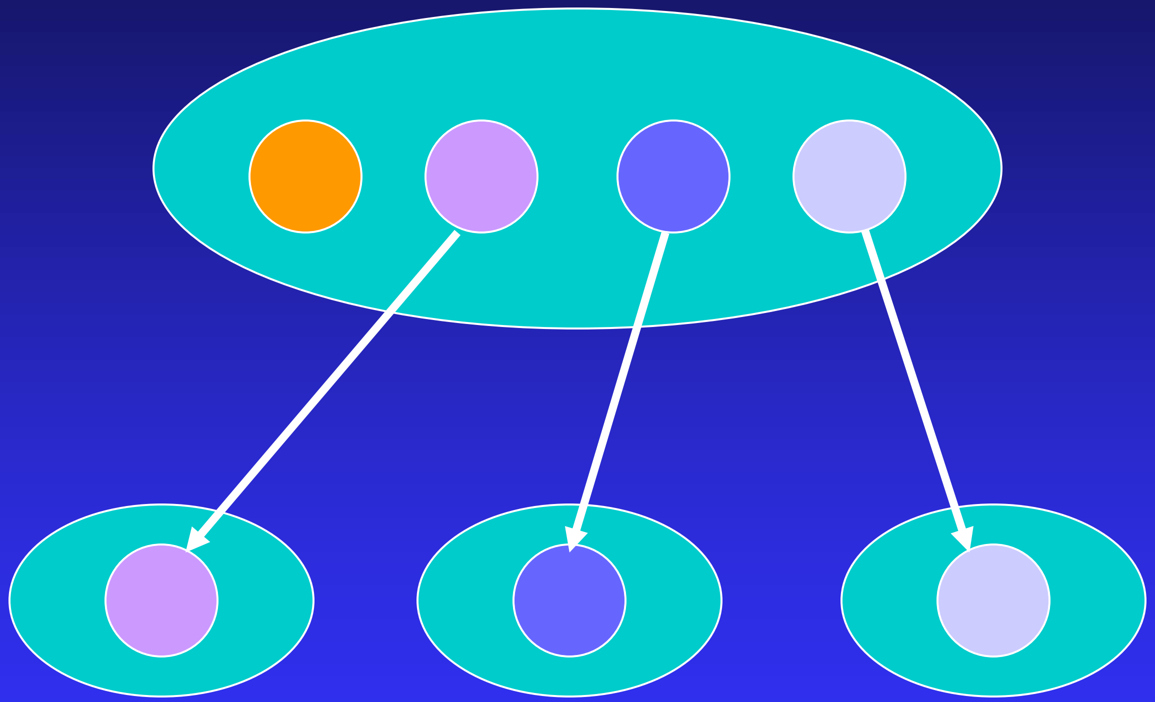
Hypercube

$$\sum_{i=1}^{\log p}\left(\lambda+\frac{2^{\text{i-1}}n}{\beta p}\right)=\lambda\log p+\frac{n(p-1)}{\beta p}$$

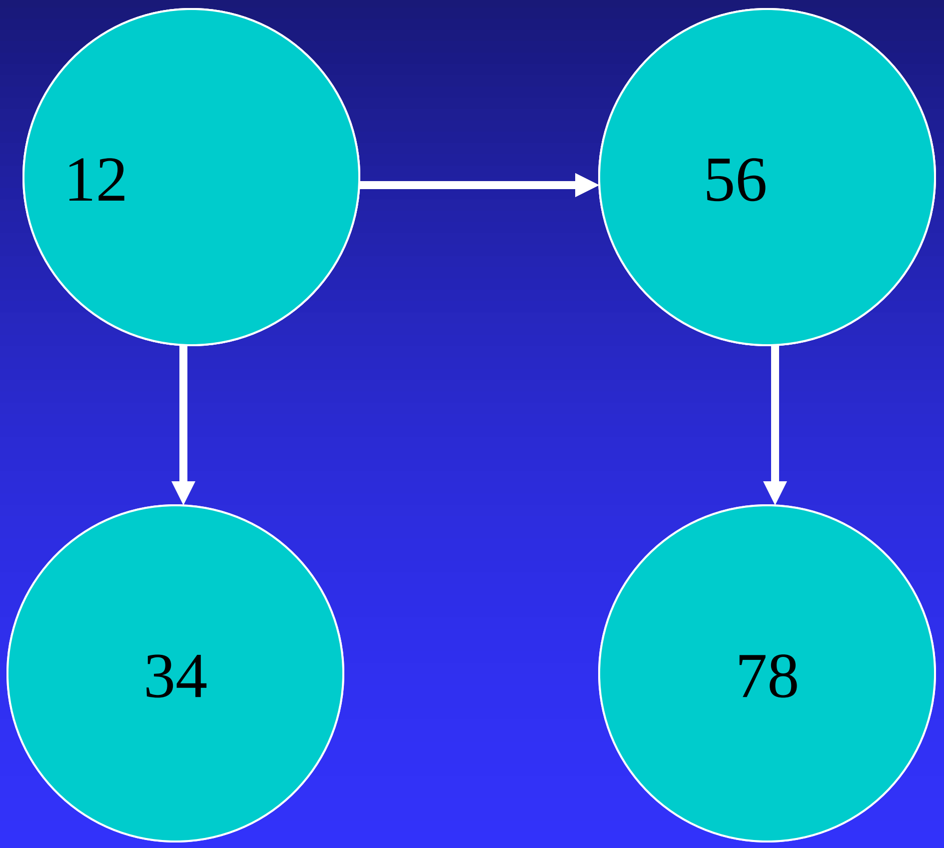
Adding Data Input



Scatter



Scatter in log *p* Steps



Summary: Task/channel Model

- Parallel computation
  - Set of tasks
  - Interactions through channels
- Good designs
  - Maximize local computations
  - Minimize communications
  - Scale up

Summary: Design Steps

- Partition computation
- Agglomerate tasks
- Map tasks to processors
- Goals
  - Maximize processor utilization
  - Minimize inter-processor communication

Summary: Fundamental Algorithms

- Reduction
- Gather and scatter
- All-gather