Parallel Programming
with MPI and OpenMP

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Chapter 7

Performance Analysis

Learning Objectives

- Predict performance of parallel programs
- Understand barriers to higher performance

Outline

- General speedup formula
- Amdahl's Law
- Gustafson-Barsis' Law
- Karp-Flatt metric
- Isoefficiency metric

Speedup Formula

$$\text{Speedup} = \frac{\text{Sequential execution time}}{\text{Parallel execution time}}$$

Execution Time Components

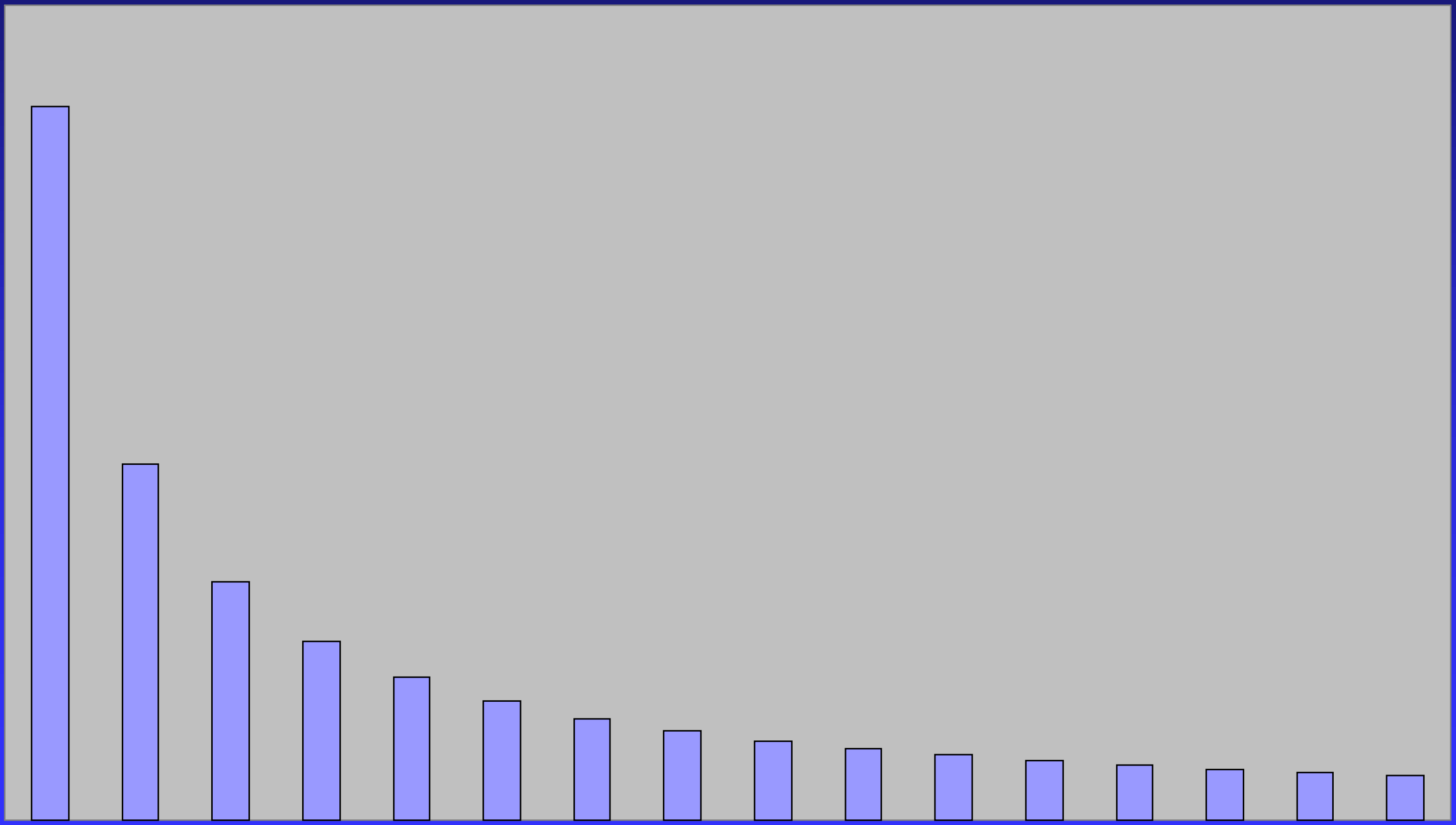
- Inherently sequential computations: $\sigma(n)$
  - *sigma*
- Potentially parallel computations: $\varphi(n)$
  - *phi*
- Communication operations: $\kappa(n,p)$
  - *kappa*

Speedup Expression

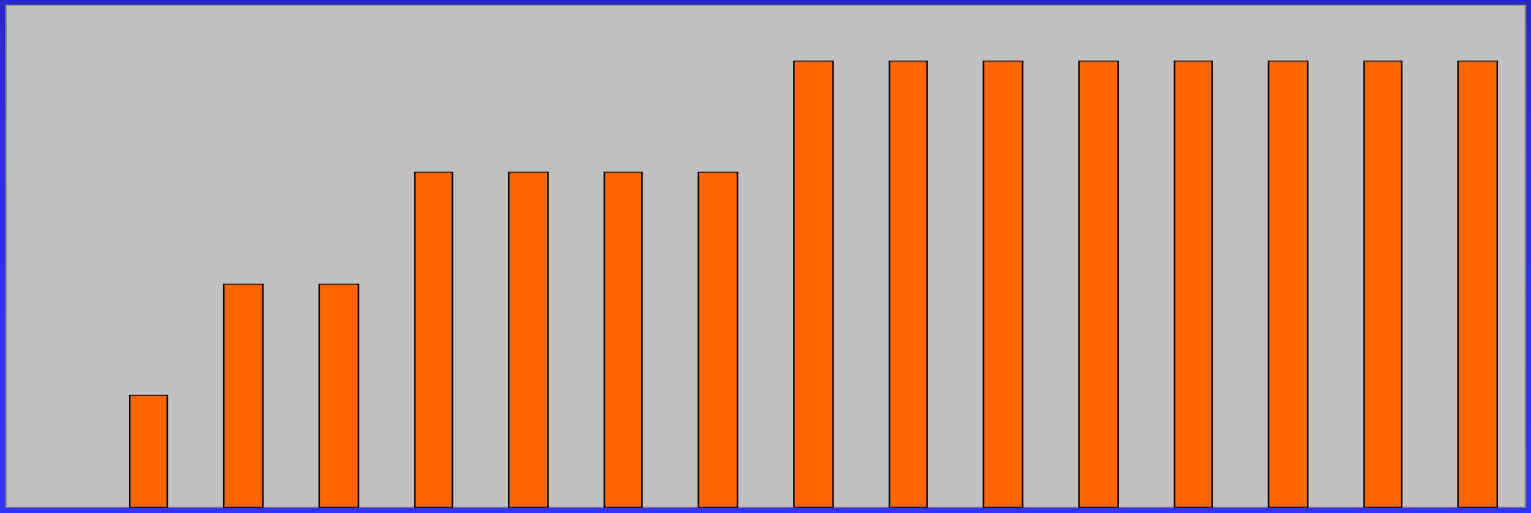
$$\psi(n,p) \le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p+\kappa(n,p)}$$

(Speedup: *si)*

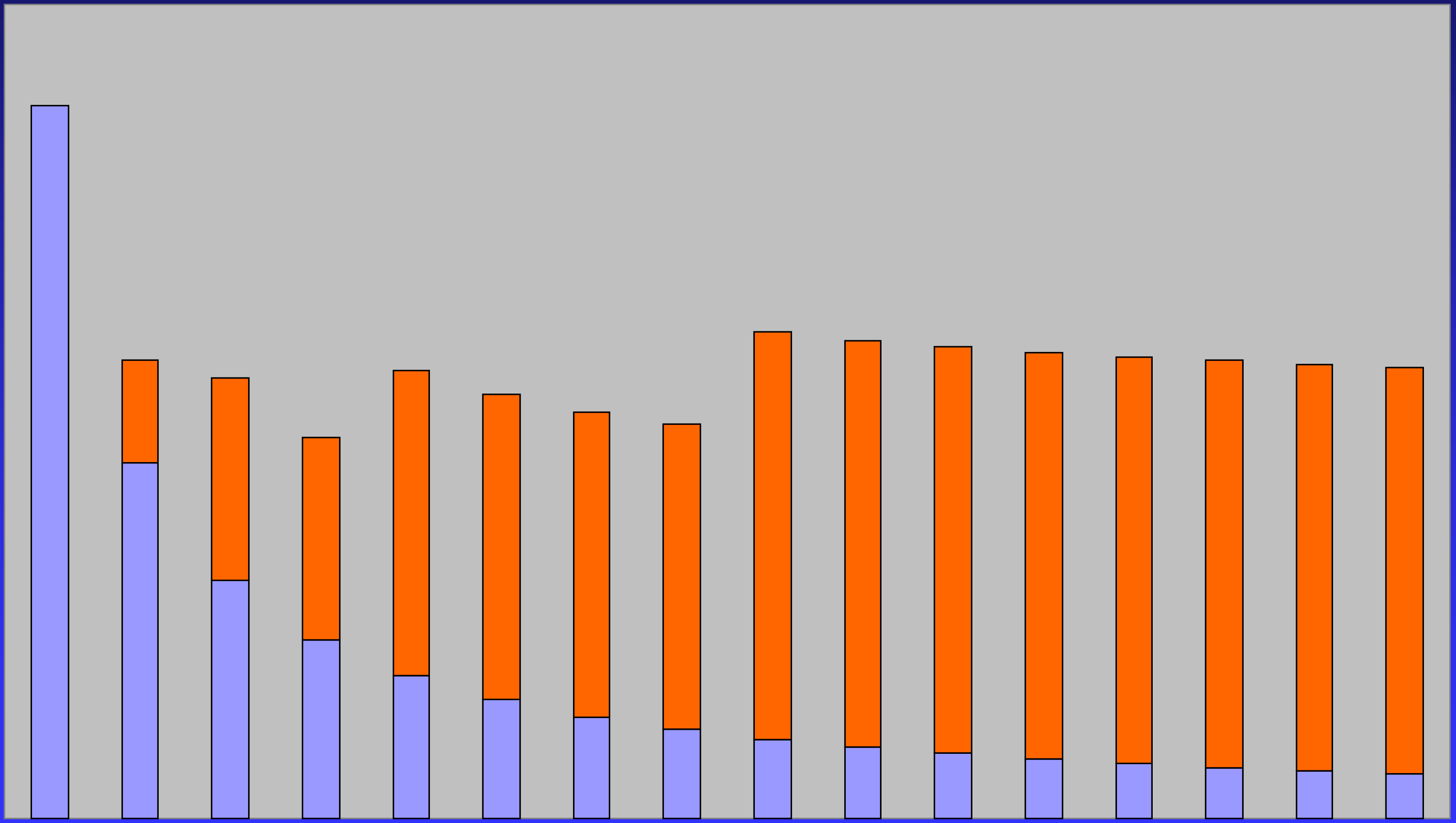
φ(n)/p



$\kappa(n,p)$

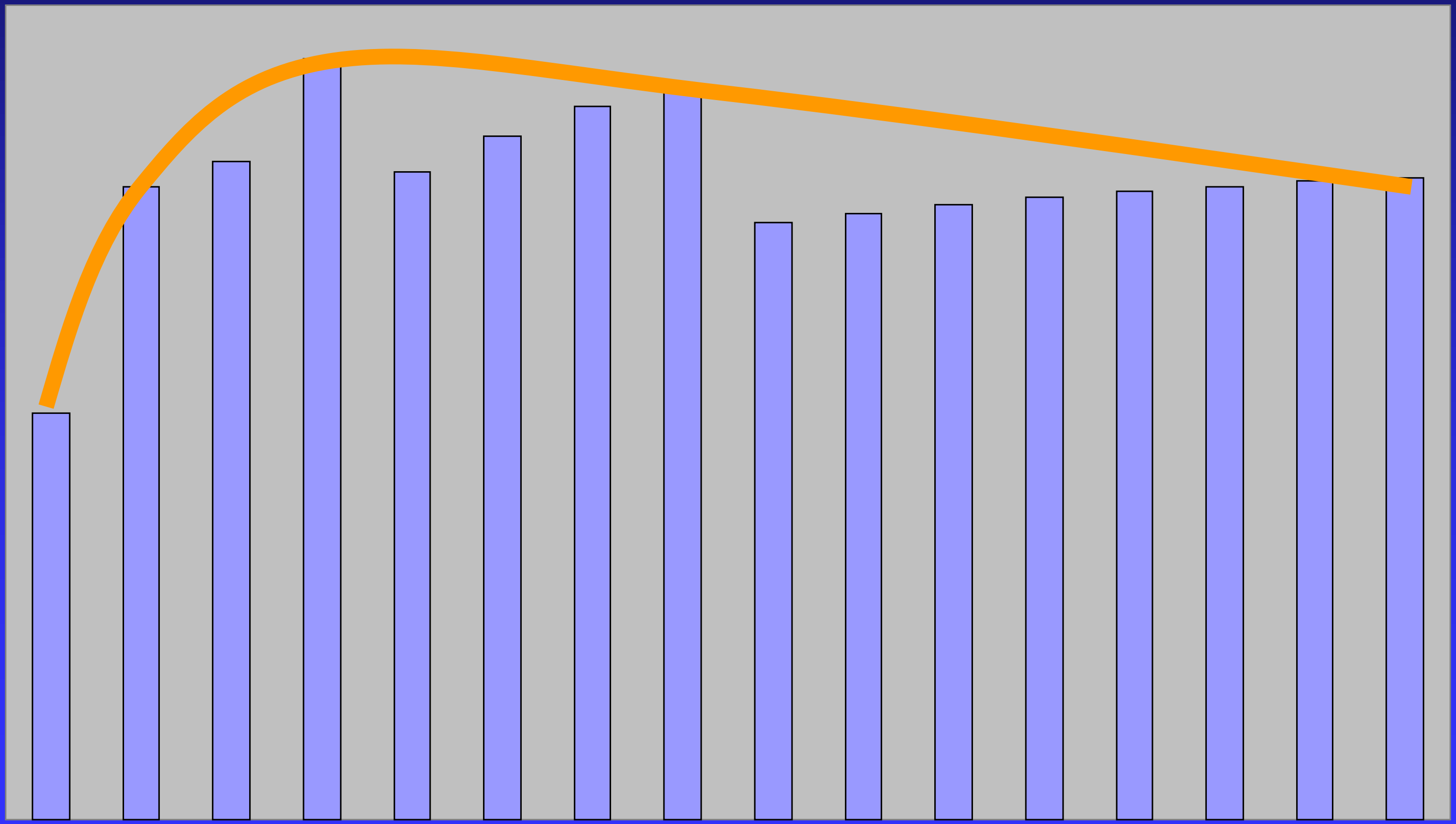


$\varphi(n)/p + \kappa(n,p)$



Speedup Plot

“elbowing out”



Efficiency

$$\text{Efficiency} = \frac{\text{Sequential execution time}}{\text{Processors} \times \text{Parallel execution time}}$$

$$\text{Efficiency} = \frac{\text{Speedup}}{\text{Processors}}$$

Efficiency is a fraction:

$$0 \le \varepsilon(n,p) \le 1 \text{ (}\mathit{Epsilon}\text{)}$$

$$\varepsilon(n, p) \le \frac{\sigma(n) + \varphi(n)}{p\sigma(n) + \varphi(n) + p\kappa(n, p)}$$

All terms > 0 $\Rightarrow \varepsilon(n,p) > 0$

Denominator > numerator $\Rightarrow \varepsilon(n,p) < 1$

Amdahl's Law

$$\psi(n,p) \le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p+\kappa(n,p)}$$

$$\le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p}$$

Let $f = \sigma(n)/(\sigma(n) + \varphi(n))$; i.e., f is the fraction of the code which is inherently sequential

$$\psi \le \frac{1}{f+(1-f)/p}$$

Example 1

- 95% of a program's execution time occurs inside a loop that can be executed in parallel. What is the maximum speedup we should expect from a parallel version of the program executing on 8 CPUs?

$$\psi \le \frac{1}{0.05 + (1 - 0.05)/8} \cong 5.9$$

Example 2

- 20% of a program's execution time is spent within inherently sequential code. What is the limit to the speedup achievable by a parallel version of the program?

$$\lim_{p\to\infty} \frac{1}{0.2+(1-0.2)/p} = \frac{1}{0.2} = 5$$

Pop Quiz

- An oceanographer gives you a serial program and asks you how much faster it might run on 8 processors. You can only find one function amenable to a parallel solution. Benchmarking on a single processor reveals 80% of the execution time is spent inside this function. What is the best speedup a parallel version is likely to achieve on 8 processors?

Pop Quiz

- A computer animation program generates a feature movie frame-by-frame. Each frame can be generated independently and is output to its own file. If it takes 99 seconds to render a frame and 1 second to output it, how much speedup can be achieved by rendering the movie on 100 processors?

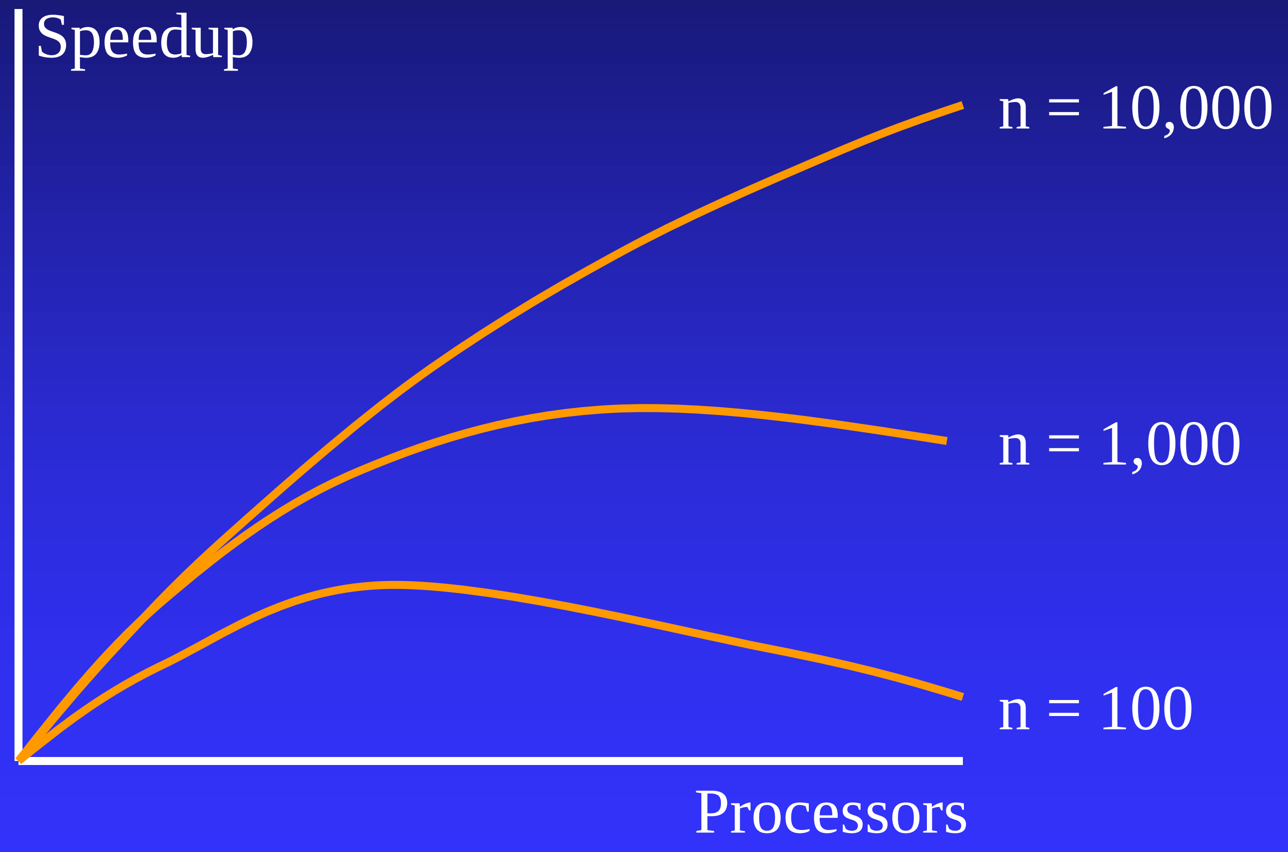
Limitations of Amdahl's Law

- Ignores $\kappa(n,p)$ - overestimates speedup
- Assumes f constant, so underestimates speedup achievable

Amdahl Effect

- Typically $\sigma(n)$ and $\kappa(n,p)$ have lower complexity than $\varphi(n)/p$
- As n increases, $\varphi(n)/p$ dominates $\sigma(n)$ & $\kappa(n,p)$
- *As n increases, speedup increases*
- *As n increases, sequential fraction f decreases.*

Illustration of Amdahl Effect



Review of Amdahl's Law

- Treats problem size as a constant
- Shows how execution time decreases as number of processors increases

Another Perspective

- We often use faster computers to solve larger problem instances
- Let’s treat time as a constant and allow problem size to increase with number of processors

Gustafson-Barsis's Law

$$\psi(n,p) \le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p}$$

Let $T_p = \sigma(n)+\varphi(n)/p = 1$ unit

Let *s* be the fraction of *time* that a parallel program spends executing the serial portion of the code.

$$s = \sigma(n)/(\sigma(n)+\varphi(n)/p)$$

Then,

$$\psi = T_1/T_p = T_1 <= s + p*(1-s)$$ (the *scaled speedup*)

Thus, sequential time would be p times the parallelized portion of the code plus the time for the sequential portion.

$$\psi \le p + (1-p)s$$

Gustafson-Barsis's Law

- ψ <= s + p*(1-s) (the *scaled speedup*)

Restated, $$\psi \le p + (1-p)s$$

Thus, sequential time would be p times the parallel execution time minus (p-1) times the sequential portion of execution time.

Gustafson-Barsis's Law

- Begin with parallel execution time and estimate the time spent in sequential portion.
- Predicts **scaled speedup (Sp - ψ - same as T_1)**
- Estimate sequential execution time to solve same problem (s)
- Assumes that s remains fixed irrespective of how large is p - thus overestimates speedup.
- Problem size (s + p*(1-s)) is an increasing function of *p*

Example 1

- An application running on 10 processors spends 3% of its time in serial code. What is the scaled speedup of the application?

$$\psi = 10 + (1 - 10)(0.03) = 10 - 0.27 = 9.73$$

…except 9 do not have to execute serial code

Execution on 1 CPU takes 10 times as long…

Example 2

- What is the maximum fraction of a program's parallel execution time that can be spent in serial code if it is to achieve a scaled speedup of 7 on 8 processors?

$$7 = 8 + (1 - 8)s \Rightarrow s \approx 0.14$$

Pop Quiz

- A parallel program executing on 32 processors spends 5% of its time in sequential code. What is the scaled speedup of this program?

The Karp-Flatt Metric

- Amdahl's Law and Gustafson-Barsis' Law ignore $\kappa(n,p)$
- They can overestimate speedup or scaled speedup
- Karp and Flatt proposed another metric

Experimentally Determined Serial Fraction

$$e = \frac{\sigma(n) + \kappa(n,p)}{\sigma(n) + \varphi(n)}$$

Inherently serial component of parallel computation + processor communication and synchronization overhead

Single processor execution time

$$e = \frac{1/\psi - 1/p}{1 - 1/p}$$

Experimentally Determined Serial Fraction

- Takes into account parallel overhead
- Detects other sources of overhead or inefficiency ignored in speedup model
  - Process startup time
  - Process synchronization time
  - Imbalanced workload
  - Architectural overhead

Example 1

p	2	3	4	5	6	7	8
ψ	1.8	2.5	3.1	3.6	4.0	4.4	4.7

What is the primary reason for speedup of only 4.7 on 8 CPUs?

e	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Since *e* is constant, large serial fraction is the primary reason.

Example 2

p	2	3	4	5	6	7	8
ψ	1.9	2.6	3.2	3.7	4.1	4.5	4.7

What is the primary reason for speedup of only 4.7 on 8 CPUs?

e	0.070	0.075	0.080	0.085	0.090	0.095	0.100

Since *e* is steadily increasing, overhead is the primary reason.

Isoefficiency Metric

- Parallel system: parallel program executing on a parallel computer
- Scalability of a parallel system: measure of its ability to increase performance as number of processors increases
- A scalable system maintains efficiency as processors are added
- Isoefficiency: way to measure scalability

Isoefficiency Derivation Steps

- Begin with speedup formula
- Compute total amount of overhead
- Assume efficiency remains constant
- Determine relation between sequential execution time and overhead

Deriving Isoefficiency Relation

Determine overhead

$$T_o(n,p) = (p-1)\sigma(n) + p\kappa(n,p)$$

Substitute overhead into speedup equation

$$\psi(n,p) \le \frac{p(\sigma(n)+\varphi(n))}{\sigma(n)+\varphi(n)+T_0(n,p)}$$

Substitute T(n,1) = σ(n) + φ(n). Assume efficiency is constant. Hence, T_0/T_1 should be a constant fraction.

$$T(n,1) \ge CT_0(n,p)$$

Isoefficiency Relation

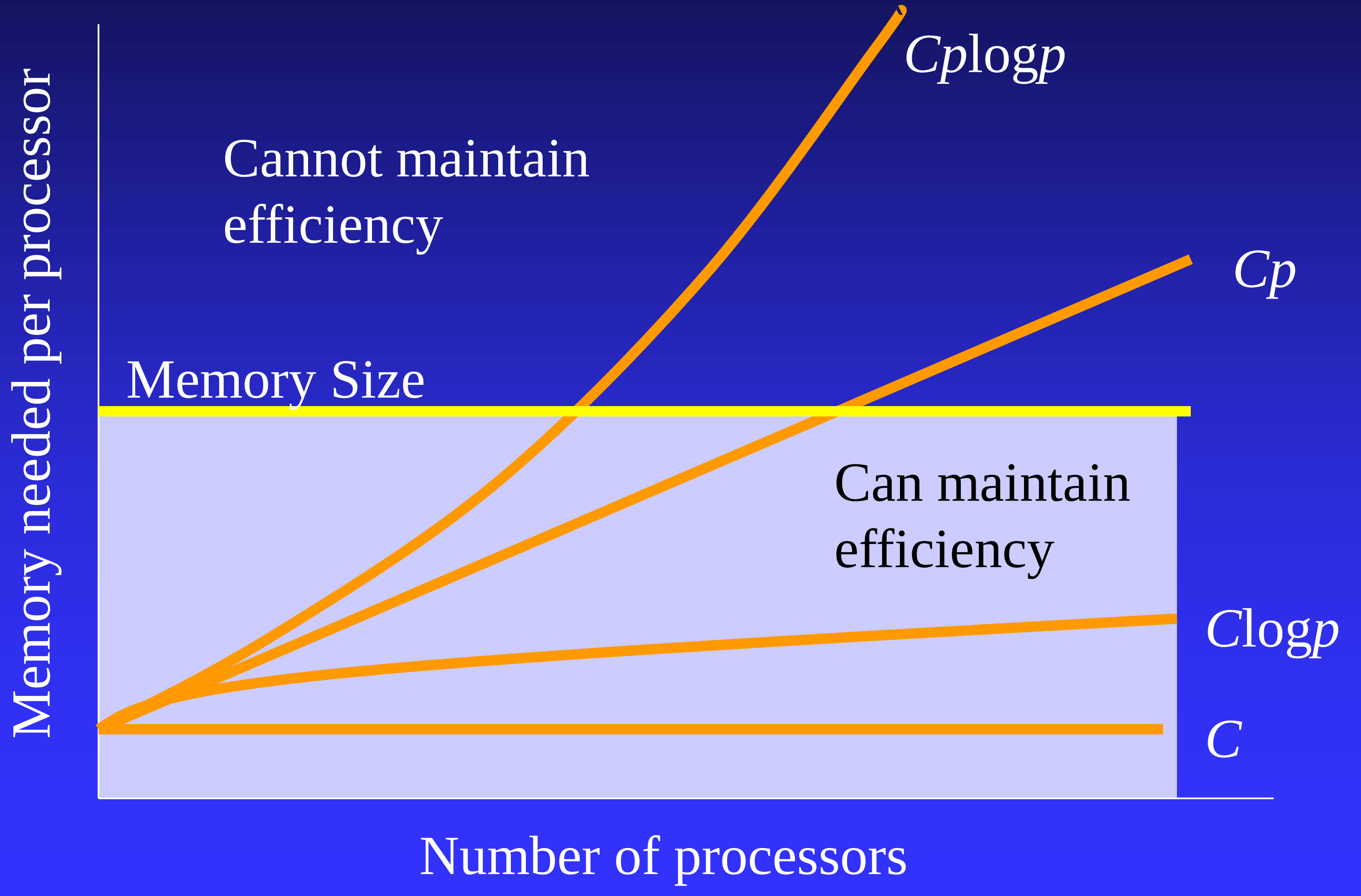
Scalability Function

- Suppose isoefficiency relation is $n \geq f(p)$
- Let $M(n)$ denote memory required for problem of size n
- $M(f(p))/p$ shows how memory usage **per processor** must increase to maintain same efficiency
- We call $M(f(p))/p$ the scalability function

Meaning of Scalability Function

- To maintain efficiency when increasing p, we must increase n
- Maximum problem size limited by available memory, which is linear in p
- Scalability function shows how memory usage per processor must grow to maintain efficiency
- Scalability function a constant means parallel system is perfectly scalable

Interpreting Scalability Function



Example 1: Reduction

- Sequential algorithm complexity
  $T(n,1) = \Theta(n)$
- Parallel algorithm
  - Computational complexity = $\Theta(n/p)$
  - Communication complexity = $\Theta(\log p)$
- Parallel overhead
  $T_0(n,p) = \Theta(p \log p)$

Reduction (continued)

- Isoefficiency relation: $n \geq C\, p \log p$
- We ask: To maintain same level of efficiency, how must n increase when p increases?
- $M(n) = n$

$$M(Cp\log p)/p = Cp\log p/p = C\log p$$

- The system has good scalability

Example 2: Floyd's Algorithm

- Sequential time complexity: $\Theta(n^3)$
- Parallel computation time: $\Theta(n^3/p)$
- Parallel communication time: $\Theta(n^2 \log p)$
- Parallel overhead: $T_0(n,p) = \Theta(pn^2 \log p)$

Floyd's Algorithm (continued)

- Isoefficiency relation
  $n^3 \geq C(p\, n^3 \log p) \Rightarrow n \geq C\, p \log p$
- $M(n) = n^2$

$$M(Cp\log p)/p = C^2 p^2 \log^2 p / p = C^2 p \log^2 p$$

- The parallel system has poor scalability

Example 3: Finite Difference

- Sequential time complexity per iteration: $\Theta(n^2)$
- Parallel communication complexity per iteration: $\Theta(n/\sqrt{p})$
- Parallel overhead: $\Theta(n\sqrt{p})$

Finite Difference (continued)

- Isoefficiency relation
  $n^2 \geq Cn\sqrt{p} \Rightarrow n \geq C\sqrt{p}$
- $M(n) = n^2$

$$M(C\sqrt{p})/p = C^2 p/p = C^2$$

- This algorithm is perfectly scalable

Summary (1/3)

- Performance terms
  - Speedup
  - Efficiency
- Model of speedup
  - Serial component
  - Parallel component
  - Communication component

Summary (2/3)

- What prevents linear speedup?
  - Serial operations
  - Communication operations
  - Process start-up
  - Imbalanced workloads
  - Architectural limitations

Summary (3/3)

- Analyzing parallel performance
  - Amdahl's Law
  - Gustafson-Barsis' Law
  - Karp-Flatt metric
  - Isoefficiency metric