1. (a) $\cos\frac{2n\pi}{3} + i\sin\frac{2n\pi}{3} = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^n =$

$= \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^n = \left(-\frac{1}{2}\right)^n + n\cdot\left(-\frac{1}{2}\right)^{n-1}\cdot\frac{i\sqrt{3}}{2} + \binom{n}{2}\left(-\frac{1}{2}\right)^{n-2}\left(\frac{i\sqrt{3}}{2}\right)^2 +$

$+ \binom{n}{3}\left(-\frac{1}{2}\right)^{n-3}\left(\frac{i\sqrt{3}}{2}\right)^3 + \binom{n}{4}\left(-\frac{1}{2}\right)^{n-4}\left(\frac{i\sqrt{3}}{2}\right)^4 +$

$+ \binom{n}{5}\left(-\frac{1}{2}\right)^{n-5}\left(\frac{i\sqrt{3}}{2}\right)^5 + \binom{n}{6}\left(-\frac{1}{2}\right)^{n-6}\left(\frac{i\sqrt{3}}{2}\right)^6 + \dots$

$\cos\frac{2n\pi}{3} = \frac{(-1)^n}{2^n} + \binom{n}{2}\frac{(-1)^n}{2^{n-2}}\cdot\left(-\frac{3}{4}\right) + \binom{n}{4}\frac{(-1)^n}{2^{n-4}}\cdot\frac{9}{2^4} +$

$+ \binom{n}{6}\frac{(-1)^n}{2^{n-6}}\cdot\frac{-27}{2^6} + \dots \quad / \cdot (-1)^n 2^n$

$(-1)^n 2^n \cos\frac{2n\pi}{3} = 1 - 3\binom{n}{2} + 9\binom{n}{4} - 27\binom{n}{6} + \dots$

(b) $(1 + 2\omega + 3\omega^2 + \dots + n\omega^{n-1})(\omega - 1) = n$?

$(1 + 2\omega + 3\omega^2 + \dots + n\omega^{n-1})(\omega - 1) = \omega + 2\omega^2 + 3\omega^3 + \dots +$

$+ n\omega^n - 1 - 2\omega - 3\omega^2 - \dots - n\omega^{n-1} = -1 - \omega - \omega^2 - \omega^3 - \dots$

$\omega^{n-1} + n\cdot\omega^n = -\frac{1-\omega^n}{1-\omega} + n\cdot\omega^n = n$

2. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{1}{(x-1)^2 + (y-2)^2}\cdot 2(y-2)$ (1)

$u = 2(y-2)\int\frac{dx}{(x-1)^2 + (y-2)^2} = \frac{2}{y-2}\int\frac{dx}{1 + \left(\frac{x-1}{y-2}\right)^2} =$ (1)

$= \left(\begin{array}{l} \frac{x-1}{y-2} = t \\ x - 1 = (y-2)t \\ dx = (y-2)\,dt \end{array}\right) = 2\int\frac{dt}{1+t^2} = 2\,\mathrm{arctg}\,\frac{x-1}{y-2} + c(y)$

(1)

$\frac{\partial u}{\partial y} = 2\cdot\frac{1}{1 + \left(\frac{x-1}{y-2}\right)^2}\cdot\left(-\frac{(x-1)}{(y-2)^2}\right) + c'(y) = -\frac{\partial v}{\partial x} =$ (1)

$= -\frac{1}{(x-1)^2 + (y-2)^2}\cdot 2(x-1)$ (1)

$\cancel{\frac{-2(x-1)}{(y-2)^2 + (x-1)^2}} + c'(y) = \cancel{\frac{-2(x-1)}{(x-1)^2 + (y-2)^2}}$

$c'(y) = 0 \Rightarrow c(y) = c_1$ (1)

$v(x,y) = 2\operatorname{arctg}\frac{x-1}{y-2} + c_1$.

$f(z) = u + iv = 2\operatorname{arctg}\frac{x-1}{y-2} + c_1 + i\ln((x-1)^2 + (y-2)^2) =$

$= \dots$ 1 1

3. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ 1

$\frac{\partial (u^2)}{\partial x} = 2u\frac{\partial u}{\partial x}$ 1 $\frac{\partial (u^2)}{\partial y} = 2u\frac{\partial u}{\partial y}$ 1

$\frac{\partial^2 (u^2)}{\partial x^2} = 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + 2u\frac{\partial^2 u}{\partial x^2} = 2\left(\frac{\partial u}{\partial x}\right)^2 + 2u\frac{\partial^2 u}{\partial x^2}$ 1

$\frac{\partial^2 (u^2)}{\partial y^2} = 2\frac{\partial u}{\partial y}\frac{\partial u}{\partial y} + 2u\frac{\partial^2 u}{\partial y^2} = 2\left(\frac{\partial u}{\partial y}\right)^2 + 2u\frac{\partial^2 u}{\partial y^2}$ 1

$\frac{\partial^2 (u^2)}{\partial x^2} + \frac{\partial^2 (u^2)}{\partial y^2} = 0$

$2\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right) + 2u\underbrace{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)}_{= 0} = 0$

$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 0 \Rightarrow \frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = 0$ 1

$u = const$ 1