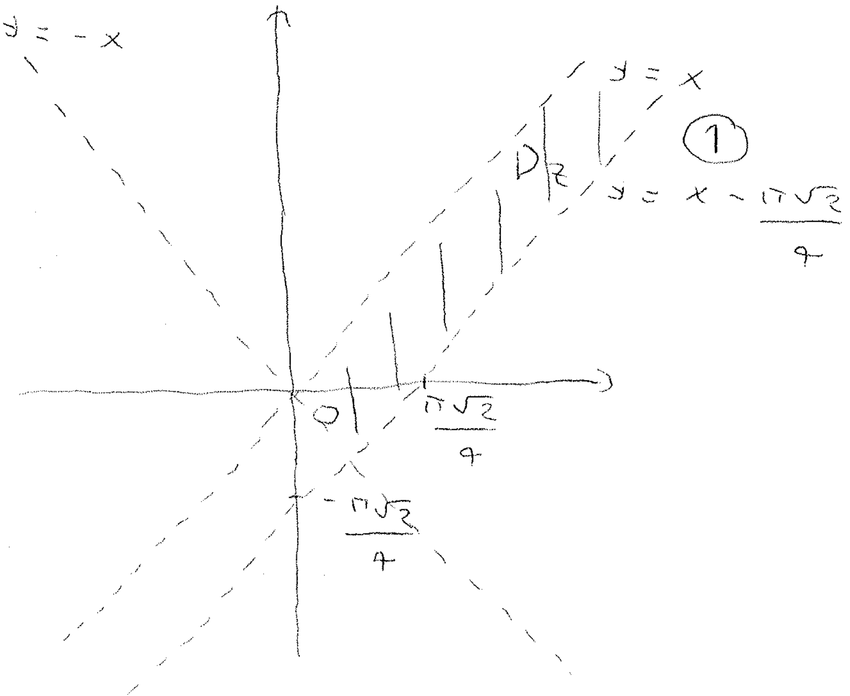
1. $w = \frac{1-e^{(1-i)z}}{1+e^{(1-i)z}} = -\frac{1+e^{(1-i)z}-2}{1+e^{(1-i)z}} = -\left(1-\frac{2}{1+e^{(1-i)z}}\right) =$

$= -1 + \frac{2}{1+e^{(1-i)z}}$



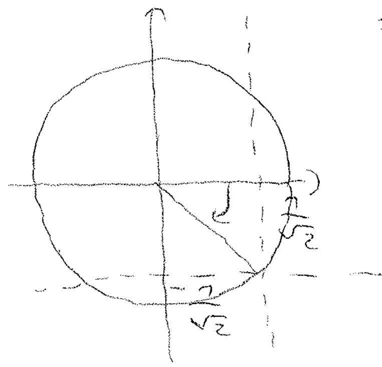
$(1-i)z = |1-i| e^{i\arg(1-i)} z = \sqrt{2} e^{i\left(-\frac{\pi}{4}\right)} z$

$|1-i| = \sqrt{1^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\cos\varphi = \frac{1}{\sqrt{2}}$

$\sin\varphi = -\frac{1}{\sqrt{2}}$

$\Rightarrow \varphi = -\frac{\pi}{4}$

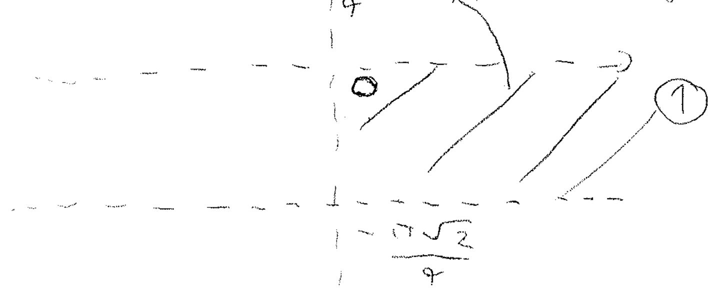


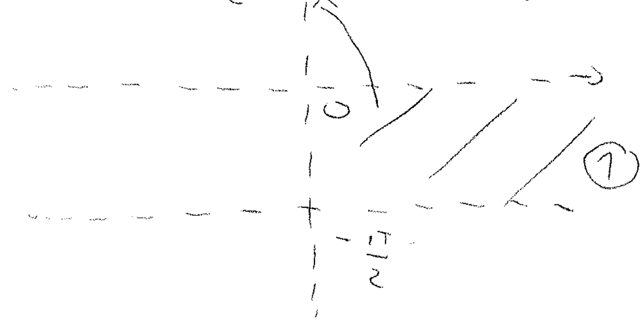
$z_1 = e^{i\left(-\frac{\pi}{4}\right)} z$

$z_2 = \sqrt{2}\, z_1$

$D_{z_1} = \{ z_1 \mid \mathrm{Re}\, z_1 > 0, -\frac{\pi\sqrt{2}}{4} < \mathrm{Im}\, z_1 < 0 \}$

$D_{z_2} = \{ z_2 \mid \mathrm{Re}\, z_2 > 0, -\frac{\pi}{2} < \mathrm{Im}\, z_2 < 0 \}.$





$z_3 = e^{z_2} = e^{x_2} \cdot e^{i y_2}$

$x_2 > 0$

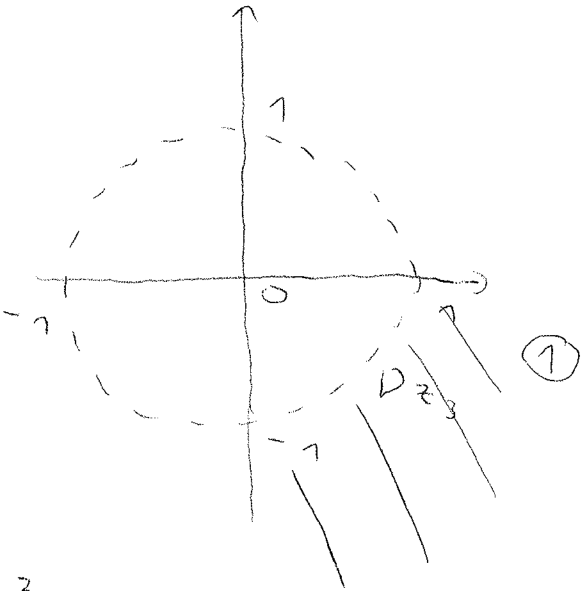
$-\frac{\pi}{2} < y_2 < 0$

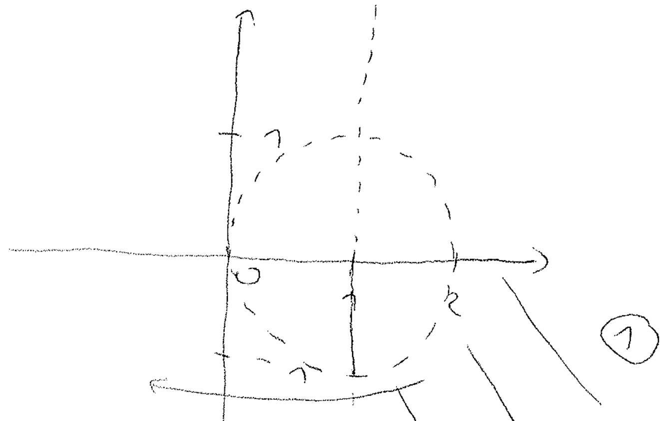
$|z_3| = e^{x_2} > 1$

$-\frac{\pi}{2} < \arg z_3 = y_2 < 0$

$D_{z_3} = \{ z_3 \mid |z_3| > 1, \ -\frac{\pi}{2} < \arg z_3 < 0 \}.$

$z_4 = 1 + z_3$





$D_{z_4} = \{ z_4 \mid \operatorname{Re} z_4 > 1, \ \operatorname{Im} z_4 < 0, \ |z_4 - 1| > 1 \}.$

$z_5 = \frac{1}{z_4}$

$z_4 = u_4 + i v_4$

$z_5 = u_5 + i v_5$

$u_4 = \frac{u_5}{u_5^2 + v_5^2}$

$v_4 = \frac{-v_5}{u_5^2 + v_5^2}$

$\operatorname{Re} z_4 > 1$

$\frac{u_5}{u_5^2 + v_5^2} - 1 > 0$

$\frac{u_5 - u_5^2 - v_5^2}{u_5^2 + v_5^2} > 0$

$u_5 - u_5^2 - v_5^2 > 0$

$u_5^2 - u_5 + v_5^2 < 0 \quad / + \frac{1}{4}$

$(u_5 - \frac{1}{2})^2 + v_5^2 < (\frac{1}{2})^2$

$\boxed{|z_5 - \frac{1}{2}| < \frac{1}{2}}$

$\operatorname{Im} z_4 < 0$

$\frac{-v_5}{u_5^2 + v_5^2} < 0$

$-v_5 < 0$

$v_5 > 0$

$\boxed{\operatorname{Im} z_5 > 0}$

$|z_4 - 1| > 1$

$(u_4 - 1)^2 + v_4^2 > 1$

$u_4^2 - 2u_4 + \cancel{1} + v_4^2 > \cancel{1}$

$\frac{1}{u_5^2 + v_5^2} - \frac{2u_5}{u_5^2 + v_5^2} > 0$

$\frac{1 - 2u_5}{u_5^2 + v_5^2} > 0$

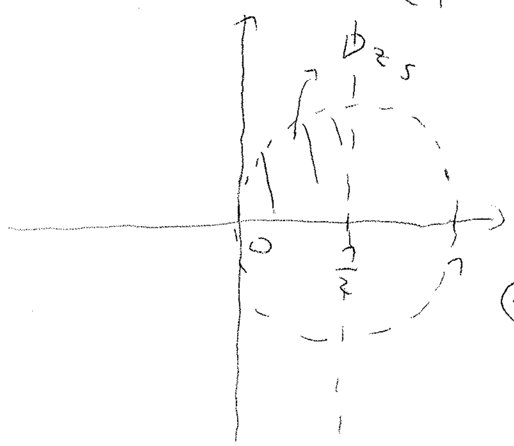
$1 - 2u_5 > 0$

$2u_5 < 1$

$u_5 < \frac{1}{2}$

$\boxed{\operatorname{Re} z_5 < \frac{1}{2}}$

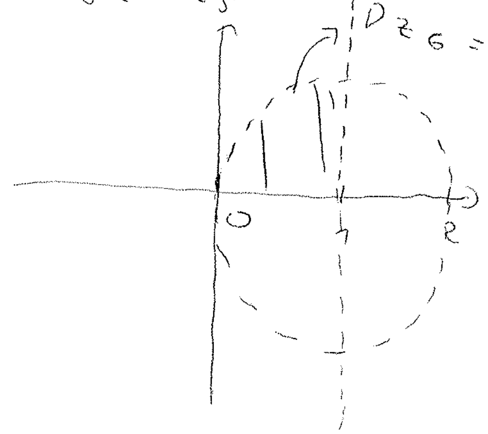
$D_{z_5} = \{ z_5 \mid |z_5 - \frac{1}{2}| < \frac{1}{2},\ \operatorname{Im} z_5 > 0,\ \operatorname{Re} z_5 < \frac{1}{2} \}.$



(1)

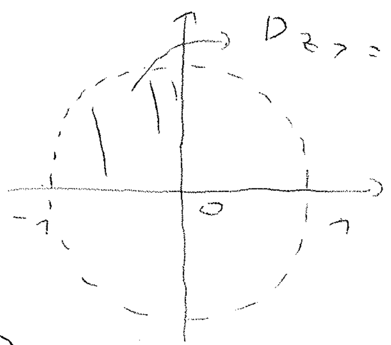
$z_6 = 2 z_5$

$D_{z_6} = \{ z_6 \mid |z_6 - 1| < 1,\ \operatorname{Re} z_6 < 1,\ \operatorname{Im} z_6 > 0 \}$



(1)

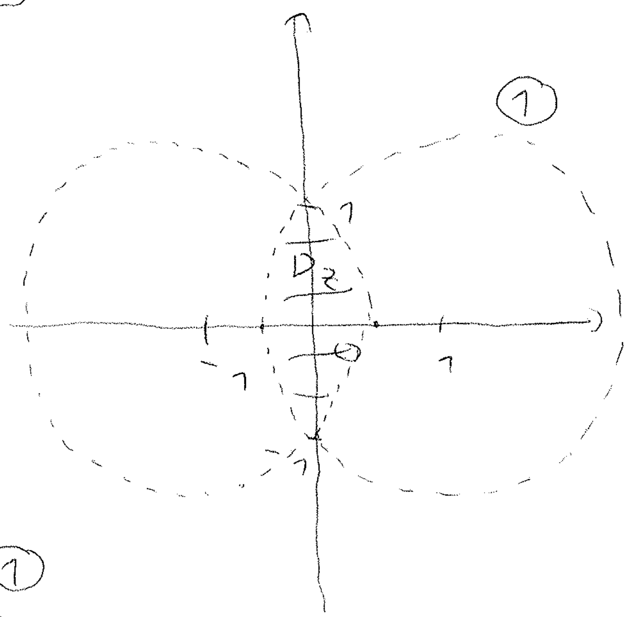
$z_7 = z_6 - 1$



$D_{z_7} = \{ z_7 \mid |z_7| < 1,\ \frac{\pi}{2} < \arg z_7 < \pi \}.$

(1)

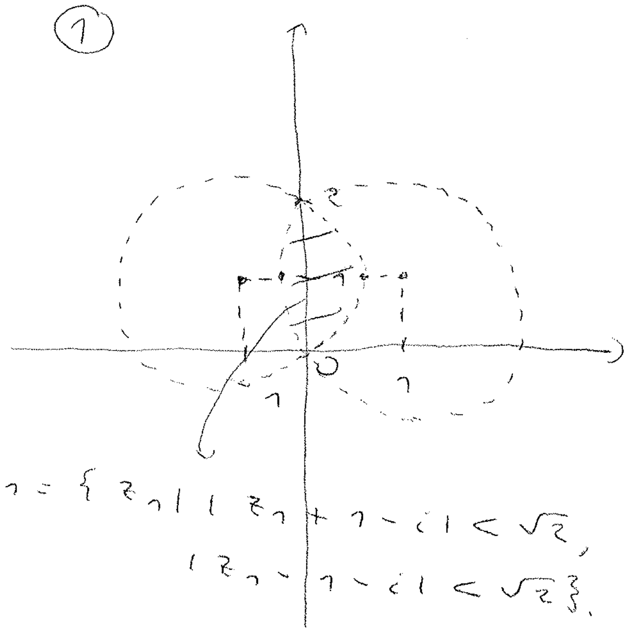
2.



(1)

$z_1 = z + i$

(1)



$D_{z_1} = \{ z_1 \mid |z_1 + 1 - i| < \sqrt{2},\ |z_1 - 1 - i| < \sqrt{2} \}.$

(1)

$z_2 = \frac{1}{z_1}$

$z_1 = u_1 + i v_1$

$z_2 = u_2 + i v_2$

$u_1 = \frac{u_2}{u_2^2 + v_2^2}$

$v_1 = \frac{-v_2}{u_2^2 + v_2^2}$

$|z_1 + 1 - i| < \sqrt{2}$

$(u_1 + 1)^2 + (v_1 - 1)^2 < 2$

$u_1^2 + 2u_1 + \cancel{1} + v_1^2 - 2v_1 + \cancel{1} < \cancel{2}$

$\frac{1}{u_2^2 + v_2^2} + \frac{2u_2}{u_2^2 + v_2^2} + \frac{2v_2}{u_2^2 + v_2^2} < 0$

$\frac{1 + 2u_2 + 2v_2}{u_2^2 + v_2^2} < 0$

$1 + 2u_2 + 2v_2 < 0$

$2v_2 < -2u_2 - 1$

$v_2 < -u_2 - \frac{1}{2}$

$|z_1 - 1 - i| < \sqrt{2}$

$(u_1 - 1)^2 + (v_1 - 1)^2 < 2$

$u_1^2 - 2u_1 + \cancel{1} + v_1^2 - 2v_1 + \cancel{1} < \cancel{2}$

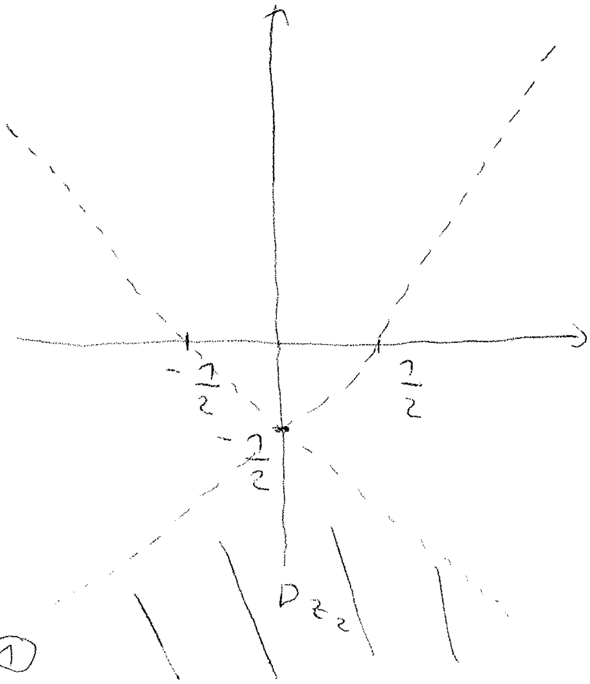
$$\frac{1}{u_2^2 + v_2^2} - \frac{2u_2}{u_2^2 + v_2^2} + \frac{2v_2}{u_2^2 + v_2^2} < 0$$

$$\frac{1 - 2u_2 + 2v_2}{u_2^2 + v_2^2} < 0$$

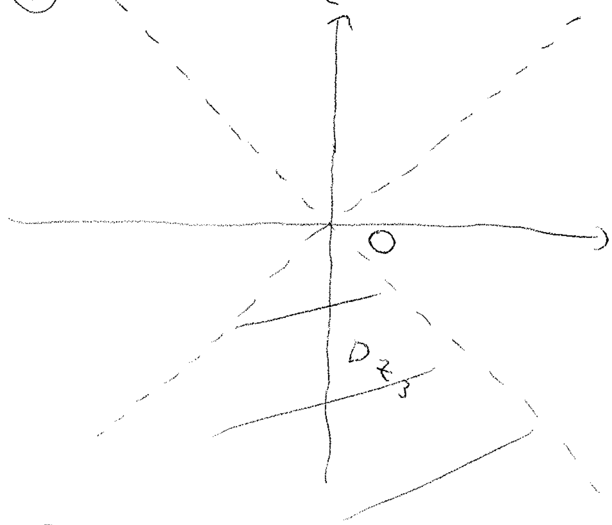
$1 - 2u_2 + 2v_2 < 0$

$2v_2 < 2u_2 - 1$

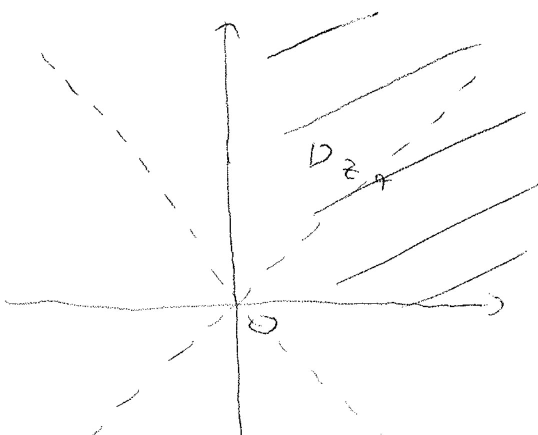
$v_2 < u_2 - \frac{1}{2}$



1 $z_3 = z_2 + \frac{i}{2}$



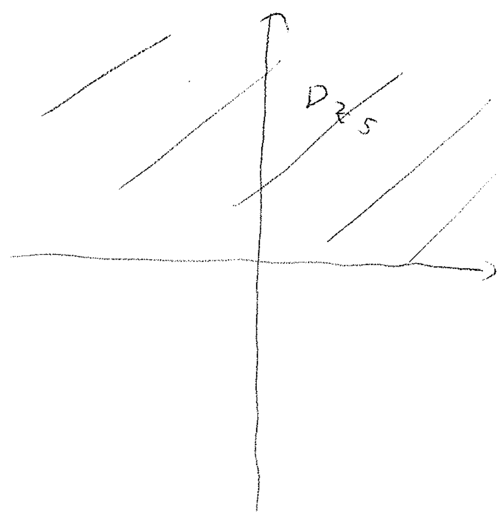
1 $z_4 = z_3 \cdot e^{i\frac{3\pi}{4}}$



1 $z_5 = z_4^2$

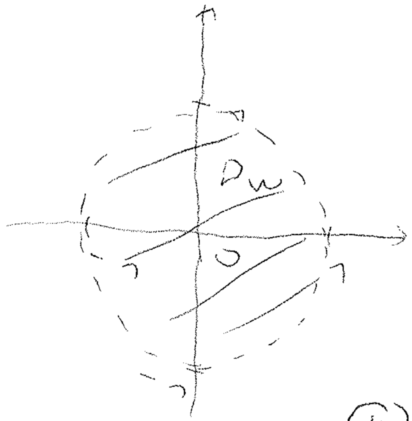
$0 < \arg z_4 < \frac{\pi}{2}$

$0 < \arg z_5 = 2 \cdot \arg z_4 < \pi$



$$w = \frac{z_5 - i}{z_5 + i}$$

(1)



3. (a) $$f(z) = \frac{1}{(z-1)^{2024}(z^2+1)} \overset{(*)}{=}$$

$$\frac{1}{z^2+1} = \frac{1}{z^2 - i^2} = \frac{1}{(z-i)(z+i)} = \frac{A}{z-i} + \frac{B}{z+i} \quad / (z-i)(z+i)$$

$$1 = A(z+i) + B(z-i)$$

$$0 = A + B \Rightarrow A = -B \qquad -Bi - Bi = 1$$

$$1 = Ai - Bi \qquad -2Bi = 1 \Rightarrow B = -\frac{1}{2i} = -\frac{1}{2i}\cdot\frac{i}{i} = \frac{i}{2}$$

$$A = -\frac{i}{2}$$

$$\frac{1}{z-i} = \frac{1}{z-1+1-i} = \frac{1}{(1-i)\left(1+\frac{z-1}{1-i}\right)} = \qquad (1)$$

$$\left|\frac{z-1}{1-i}\right| < 1 \qquad |z-1| < \sqrt{2}$$

$$|z-1| < |1-i| \qquad |1| < \sqrt{2}$$

$$1 < \sqrt{2} \checkmark$$

$$= \frac{1}{1-i}\cdot\sum_{n=0}^{+\infty}(-1)^n\left(\frac{z-1}{1-i}\right)^n = \sum_{n=0}^{+\infty}(-1)^n\frac{(z-1)^n}{(1-i)^{n+1}}. \qquad (1)$$

$$\frac{1}{z+i} = \frac{1}{z-1+1+i} = \frac{1}{(1+i)\left(1+\frac{z-1}{1+i}\right)} =$$

$$\left|\frac{z-1}{1+i}\right| < 1 \qquad |z-1| < \sqrt{2}$$

$$|z-1| < |1+i| \qquad |1| < \sqrt{2}$$

$$1 < \sqrt{2} \checkmark$$

$$= \frac{1}{1+i}\sum_{n=0}^{+\infty}(-1)^n\left(\frac{z-1}{1+i}\right)^n = \sum_{n=0}^{+\infty}(-1)^n\frac{(z-1)^n}{(1+i)^{n+1}} \qquad (1)$$

$$\overset{(*)}{=} \frac{1}{(z-1)^{2024}}\cdot\frac{i}{2}\left(\sum_{n=0}^{+\infty}(-1)^n\frac{(z-1)^n}{(1+i)^{n+1}} - \sum_{n=0}^{+\infty}(-1)^n\frac{(z-1)^n}{(1-i)^{n+1}}\right)$$

$$= \frac{i}{2} \cdot \sum_{n=0}^{+\infty} (-1)^n \left(\frac{1}{(1+i)^{n+1}} - \frac{1}{(1-i)^{n+1}} \right) (z-1)^{n-2024}$$

$[n - 2024 = k]$

$$= \sum_{k=-2024}^{+\infty} \frac{i}{2} (-1)^k \left(\frac{1}{(1+i)^{k+2025}} - \frac{1}{(1-i)^{k+2025}} \right) (z-1)^k \qquad (1)$$

(6) $$f(z) = \frac{\sin((z+1)i)}{e^{z+1}} = \frac{e^{-z-1} - e^{z+1}}{2i \cdot e^{z+1}} = \qquad (2)$$

$$= \frac{1}{2i} \cdot \frac{1}{e^{2z+2}} - \frac{1}{2i} = \frac{1}{2i} \cdot \frac{1}{e^{2z-2+4}} - \frac{1}{2i} =$$

$$= \frac{1}{2i} \cdot \frac{1}{e^4} \cdot \frac{1}{e^{2(z-1)}} - \frac{1}{2i} = \frac{1}{2ie^4} \sum_{n=0}^{+\infty} \frac{(-1)^n 2^n (z-1)^n}{n!} - \frac{1}{2i} \qquad (1)$$

$e^{-2(z-1)}$

$$= \frac{1}{2ie^4} - \frac{1}{2i} + \sum_{n=1}^{+\infty} \frac{(-1)^n 2^{n-1}}{ie^4 n!} (z-1)^n \qquad (1)$$

$$\frac{1}{2i} \cdot \frac{1 - e^4}{e^4}$$

$, \ x \in \mathbb{R}$